Section 3.2: The Exponential Function

Derivatives of Exponential Functions:

In the worksheet you completed, you concluded that the derivative of the function $f(x) = e^x$ is given by $f'(x) = e^x$. Part of the calculation came down to using the definition of the derivative to conclude that

$$f'(x) = e^x \cdot \lim_{h \to 0} \frac{e^h - 1}{h}.$$

The conclusion then came down to the fact that

$$\lim_{h \to 0} \frac{e^h - 1}{h} = 1.$$

In a similar way, we can study the derivative of the function $f(x) = a^x$, where $a > 0$, $a \neq 1$. An identical calculation to the one in the worksheet shows that

$$f'(x) = a^x \cdot \lim_{h \to 0} \frac{a^h - 1}{h}.$$

It is possible to show that

$$\lim_{h \to 0} \frac{a^h - 1}{h} = \ln a.$$

Thus, we are led to the following conclusion:

> DERIVATIVE OF AN EXPONENTIAL FUNCTION: If $a > 0$ and $a \neq 1$, we have
> $$\frac{d}{dx}(a^x) = \ln a \cdot a^x.$$

Question: Is the above formula consistent with $\dfrac{d}{dx}(e^x) = e^x$?

Examples:

1. Find the derivative of the following functions.

   (a) $f(x) = 12e^x + 11^x$

   (b) $y = 2^x + \dfrac{2}{x^3}$

   (c) $h(z) = (\ln 2)^z$

   (d) $f(x) = \pi^2 + \pi^x$

   (e) $f(x) = e^{1+x}$

2. Using an equation of the tangent line to the graph of e^x at $x = 0$, show that

$$e^x \geq 1 + x.$$