# Section 3.3: The Product and Quotient Rules

Today we will learn how to take derivatives of functions of the form $f(x)g(x)$ and $\dfrac{f(x)}{g(x)}$. We will begin by trying to use the limit definition of the derivative to understand how to take the derivative of the product of two functions.

**Problem:** Suppose that $f(x)$ and $g(x)$ are both differentiable functions, and $P(x) = f(x)g(x)$. Use the limit definition of the derivative to write an expression for $P'(x)$ in terms of $f$ and $g$:

$$P'(x) = \lim_{h \to 0} \frac{P(x+h) - P(x)}{h}$$

Now, add the term $f(x)g(x+h)$ and subtract the same term in the numerator (i.e. add zero!) so that you are able to complete the limit and come up with an expression for $P'(x)$ in terms of $f(x)$, $f'(x)$, $g(x)$, and $g'(x)$:

THE PRODUCT RULE: If $f(x)$ and $g(x)$ are both differentiable functions, we have

$$\frac{d}{dx}[f(x)g(x)] = f'(x)g(x) + f(x)g'(x).$$

**Examples:**

1. Find the derivative of the following functions.

   (a) $y = x \cdot 2^x$

   (b) $y = (t^2 + t)e^t$

   (c) $y = (t^3 - 7t^2 + 1)e^t$

THE QUOTIENT RULE: If $f(x)$ and $g(x)$ are both differentiable functions, we have

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$$

2. Find the derivative of the following functions.

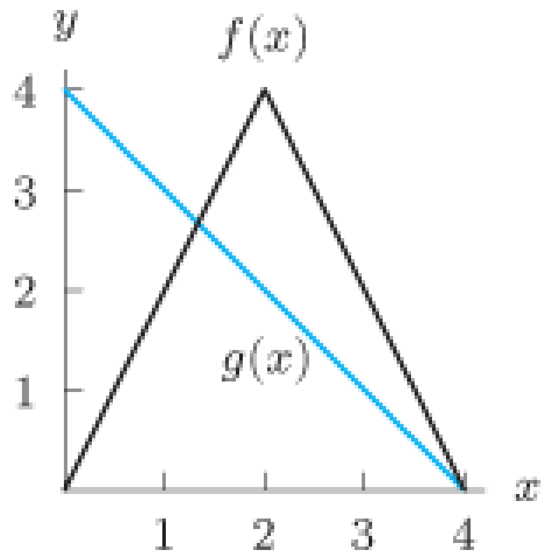
(a) $g(x) = \dfrac{25x^2}{e^x}$

(b) $g(t) = \dfrac{t-4}{t+4}$

(c) $z = \dfrac{t^2 + 5t + 2}{t+3}$

(d) $h(p) = \dfrac{1+p^2}{3+2p^2}$

3. Consider the figure below. Let $h(x) = \dfrac{f(x)}{g(x)}$ and find the following:




(a) $h'(1)$

(b) $h'(2)$

(c) $h'(3)$

4. For what intervals is $f(x) = xe^x$ concave up?

5. Find an equation of the tangent line to $f(x) = \dfrac{3x^2}{5x^2 + 7x}$ at $x = 1$.

6. The quantity, $q$, of a certain skateboard solds depends on the selling price, $p$, in dollars, so we write $q = f(p)$. You are given that $f(140) = 15,000$ and that $f'(140) = -100$.

   (a) What do $f(140) = 15,000$ and $f'(140) = -100$ tell you about the sales of skateboards?

   (b) The total revenue, $R$, earned by the sale of skateboards is given by $R = pq$. Find $\left.\dfrac{dR}{dp}\right|_{p=140}$.