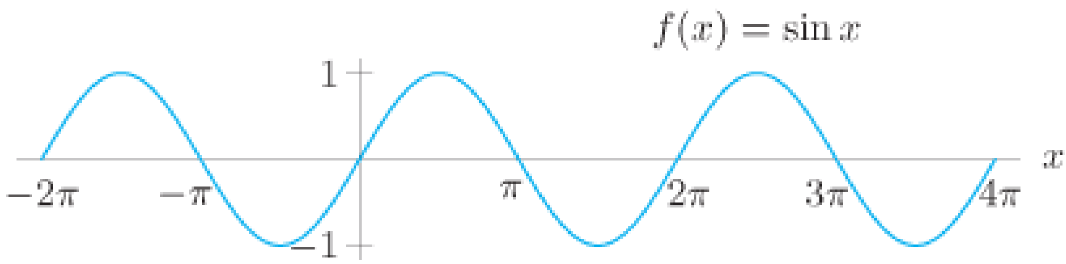
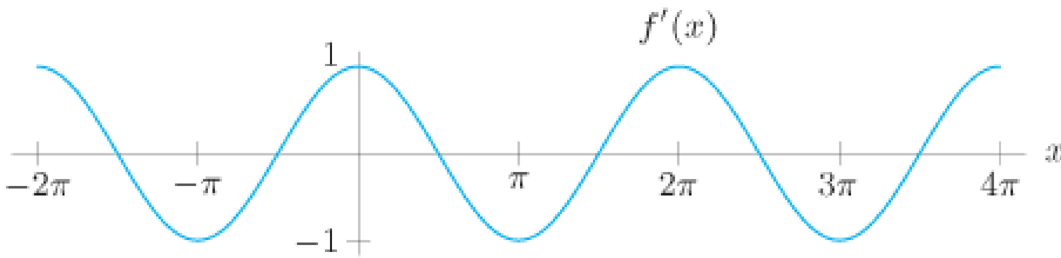
Section 3.5: The Trigonometric Functions

Consider the function $y = \sin x$, shown in the figure below.



Using the graphical techniques we have learned, we can sketch a good picture of what the derivative of this graph should like like.



Question: Based on the above two graphs, what do you expect the derivative of $\sin x$ to be equal to?

DERIVATIVE OF $\sin x$: If x is measured in radians, we have

$$\frac{d}{dx}(\sin x) = \cos x.$$

Problem: Using the relationship $\cos x = \sin\left(x + \dfrac{\pi}{2}\right)$, find the derivative of $\cos x$.

We have arrived at another important result.

DERIVATIVE OF $\cos x$: If x is measured in radians, we have

$$\frac{d}{dx}(\cos x) = -\sin x.$$

Examples:

1. Find the derivatives of the following functions.

   (a) $y = 5\sin(3t)$

   (b) $g(\theta) = \sin^2(5\theta) - \pi\theta$

   (c) $f(x) = e^{\cos x}$

   (d) $f(x) = x^2 \cos x$

The following is a list of all of the trigonometric functions and their derivatives:

DERIVATIVES OF THE TRIGONOMETRIC FUNCTIONS:

- $\dfrac{d}{dx}(\sin x) = \cos x$
- $\dfrac{d}{dx}(\cos x) = -\sin x$
- $\dfrac{d}{dx}(\tan x) = \sec^2 x = \dfrac{1}{\cos^2 x}$
- $\dfrac{d}{dx}(\csc x) = -\csc x \cot x$
- $\dfrac{d}{dx}(\sec x) = \sec x \tan x$
- $\dfrac{d}{dx}(\cot x) = -\csc^2 x = -\dfrac{1}{\sin^2 x}$

2. Find the derivatives of the following functions.

(a) $g(t) = t^2 e^{\sec t}$

(b) $f(t) = \tan(\sqrt{t^2 + 1})$

(c) $y = \csc^4(x + 3)$

(d) $W = (\tan^2 q + 5q)^3$

3. Find an equation for the tangent line to $f(t) = 3\sin(2t) + 5$ at the point where $t = \pi$.

4. The depth, y, of Boston Harbor is given in terms of the number of hours since midnight, t, by

$$y = 5 + 4.9\cos\left(\frac{\pi t}{6}\right).$$

(a) Find $\frac{dy}{dt}$. What does $\frac{dy}{dt}$ represent in terms of the water level?

(b) For $0 \leq t \leq 24$, When is $\frac{dy}{dt} = 0$? Explain what it means for $\frac{dy}{dt}$ to be zero.