Section 3.7: Implicit Functions

If $y = f(x)$, we say that y is an *explicit* function of x. However, it will often be convenient to deal with functions that are given *implicitly*, such as

$$x^2 + y^2 = 4.$$

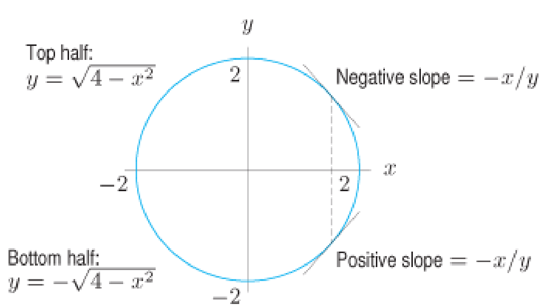
The above equation is said to give y as an *implicit* function of x.

Concept Review: Explain why y from the above equation is not actually a function of x in the same way that we understand functions.

Regardless of the fact that y is not an explicit function of x, we can still find the slope of any tangent line to the curve given by

$$x^2 + y^2 = 4$$

by considering y as an implicit function of x and using the chain rule. To do this, we take the derivative of both sides of the equation *with respect to* x and then solve for dy/dx:



Examples:

1. For the following exercises, find dy/dx.

   (a) $x^2 + y^2 + 3x - 5y = 25$

   (b) $x^2y - 2y + 5 = 0$

(c) $x \ln y + y^3 = \ln x$

(d) $e^{\cos x} = x^3 \arctan y$

2. Find the slope of the tangent line to the curve $x^3 + 2xy + y^2 = 4$ at the point $(1, 1)$.

3. Find an equation of the tangent line to

$$y^2 = \frac{x^2}{xy - 4}$$

at the point $(4, 2)$.

4. Find an equation to the tangent line to the curve $y = x^2$ at $x = 1$. Show that this line is also tangent to a circle centered at $(8, 0)$ and find an equation of this circle.