Section 4.3: Optimization and Modeling

In this section we will utilize the techniques we learned in the last couple of sections to find maximum and minimum values of quantities that might arise naturally in modeling problems. We will start with the classical introductory example.

Examples:

1. What are the dimensions of an aluminum can that holds 40 in^3 of juice and that uses the least amount of material? Assume that the can is cylindrical, and that it is capped on both ends.

Practical Tips for Modeling Optimization Problems:

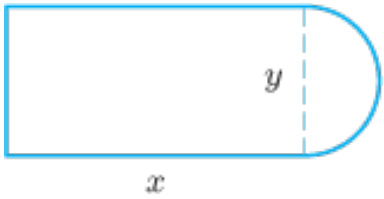
1. Make sure that you know what quantity or function is to be optimized.

2. If possible, make several sketches showing how the elements that vary are related. Label your sketches clearly by assigning variables to quantities that change.

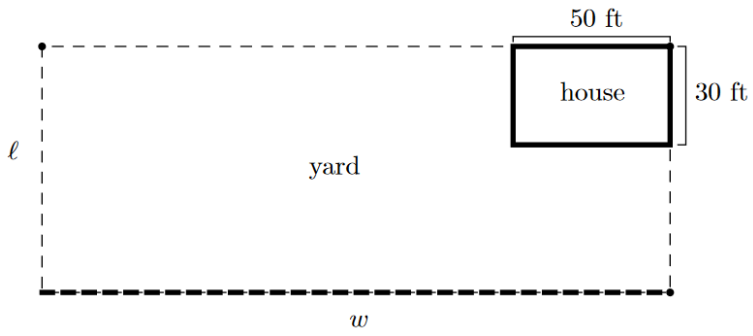
3. Try to obtain a formula for the function to be optimized in terms of the variables you identified in the previous step. If necessary, eliminate from this formula all but one variable. Identify the domain over which this variable varies.

4. Find the critical points and evaluate the function at these points and the endpoints (if relevant) to find the global maxima and/or minima.

2. Find the dimensions that maximize the area of the figure shown below given that the perimeter is 100.



3. Jason has a 50 ft by 30 ft house and wants to enclose a yard with a fence to keep his dogs in, as shown below. On the south side of his yard, he plans for the fence to be extra tall to shade his yard from the sun. Note that the fence does not extend around the sides of Jason's house. The extra tall fence (thick dashed line) costs \$15 per foot, and the rest of the fence (thin dashed line) costs \$5 per foot. Jason is planning on spending \$4500 on his fence.



Find the dimensions of the fence that maximize the area that Jason can enclose on his budget.

4. On the same side of a straight river are two towns, and the townspeople want to build a pumping station, S. The pumping station is to be at the river's edge with pipes exending straight into the two towns. Where should the pumping station be located to minimize the total length of pipe?