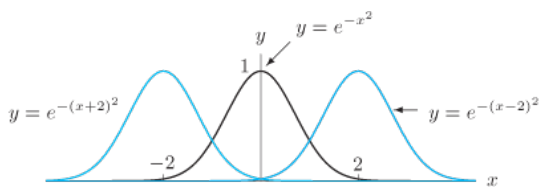
Section 4.4: Families of Functions

A *family of functions* is a function that changes depending on the particular values of certain parameters. An example of such a family of functions would be something like $f(x) = a(x-b)^2 + c$. Depending on the values of a, b, and c, this function could take multiple forms. The collection of all such functions is referred to as a family.

Example: The Bell-Shaped Curve $y = e^{-(x-a)^2/b}$ The bell-shaped curve is incredibly useful in probability and statistics, as the same shape is used to describe the *standard normal distribution*.

First we let $b = 1$ and examine the shape of the graph $y = e^{-(x-a)^2}$.

(a) Find the global maximum of $y = e^{-(x-a)^2}$ on the interval $(-\infty, \infty)$.

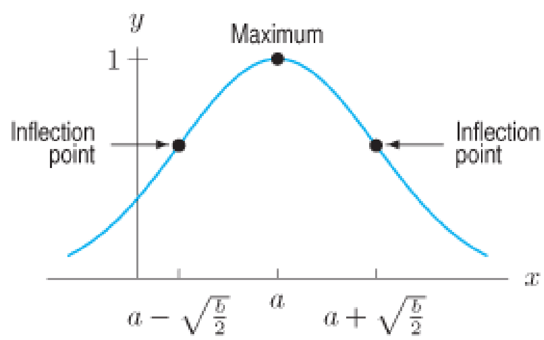


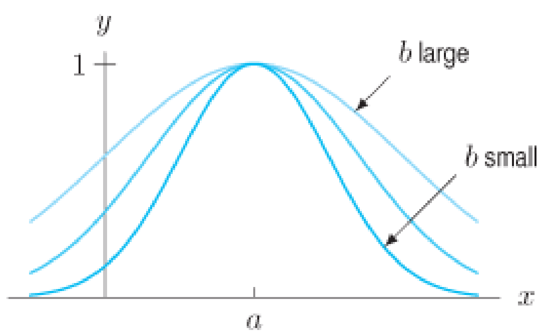
Next we let $a = 0$ and examine the graph of $y = e^{-x^2/b}$.

(b) Find the inflection points of $y = e^{-x^2/b}$.

In the previous example, we saw that the inflection points of $y = e^{-x^2/b}$ are at $x = \pm\sqrt{b/2}$. In general, for the two parameter family of functions $y = e^{-(x-a)^2/b}$, the inflection points are located at

$$x = a \pm \sqrt{\frac{b}{2}}.$$



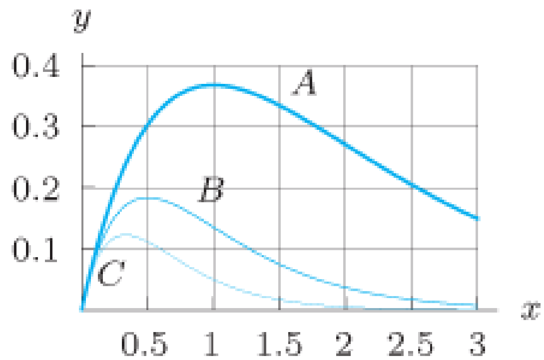


Examples:

1. If A and B are positive constants, find all critical points of

$$f(w) = \frac{A}{w^2} - \frac{B}{w}.$$

2. The graphs of $f(x) = xe^{-ax}$ for $a = 1$, 2, and 3 are pictured below. Without a calculator, identify the graphs by locating the critical points of $f(x)$.



3. Let $h(x) = e^{-x} + kx$, where k is any constant. For what value(s) of k does h have no critical points? One critical point? A horizontal asymptote?

4. Find a function of the form $y = \dfrac{a}{1 + be^{-t}}$ with y-intercept 2 and an inflection point at $t = 1$.