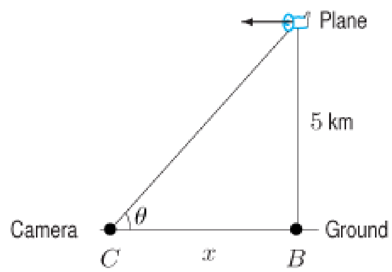
Section 4.6: Rates and Related Rates

In this section we will study a new type of word problem. We will take the information given to us in the problem and use it to find a relationship between the relevant quantities. We will then find a relationship between the rates of change (derivatives with respect to time) of the relevant quantities. We will just kick it off with some examples.

Examples:

1. A 3-meter ladder stand up against a high wall. The foot of the ladder moves outward at a constant speed of 0.1 meters per second. When the foot of the ladder is 1 meter from the wall, how fast is the top of the ladder falling? What about when the foot is 2 meters from the wall?

2. An airplane, flying at 450 km/hr at a constant altitude of 5 km, is approaching a camera mounted on the ground. Let θ be the angle of elevation above the ground at which the camera is pointed. When $\theta = \pi/3$, how fast does the camera have to rotate to keep the plane in view?



3. Grit, which is spread on roads in the winter, is stored on mounds which are the shape of a cone. As grit is added to the top of the mound at 2 cubic meters per minute, the angle between the slant side of the cone and the vertical remains at a constant $45°$. How fast is the height of the mound increasing when it is half a meter high?

4. A cone-shaped coffee filter of radius 6 cm and depth 10 cm contains water, which drips out through a hole at the bottom at a constant rate of 1.5 cm^3 per second.

   (a) If the filter starts out full, how long does it take to empty?

   (b) Find the volume of water in the filter when the depth of the water is h cm.

   (c) How fast is the water level falling when the depth is 8 cm?