Section 5.1: How Do We Measure Distance?

Just to start, let me first say that the answer to the question posed in the title of this section is *not* "measure it with a ruler". That is, of course, an acceptable way to measure distance. However, in this section we will primarily be concerned with how one would go about measuring a distance travelled if the only information one had access to was information regarding the *speed*, or *velocity*.

Let's kick this off with a thought experiment. This is the example presented in the textbook, so hopefully this doesn't come off as too boring. However, I feel that this is a perfect way to try and gain conceptual insight into the fundamentals of this chapter.

A Thought Experiment:

Suppose that a car is traveling down the road and that its speed is being recorded every two seconds. The following table shows the data from the first 10 seconds of the trip.

Time (sec)	0	2	4	6	8	10
Velocity (ft/sec)	20	30	38	44	48	50

Use the data in the table above to estimate how far the car traveled over the first 10 seconds.

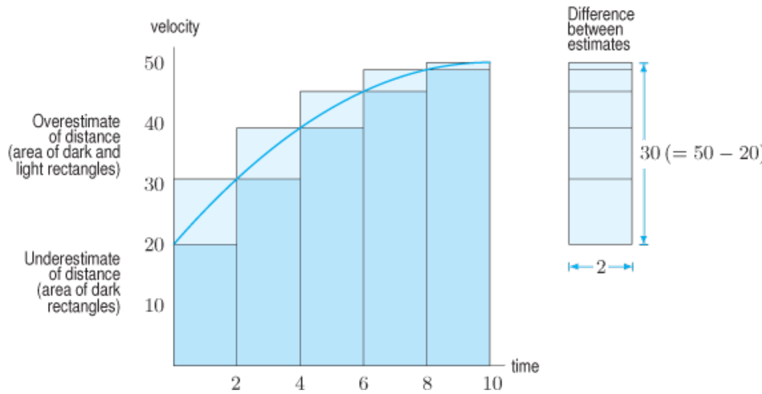
What if we increased the frequency with which we measure the velocity? Consider the following table:

Time (sec)	0	1	2	3	4	5	6	7	8	9	10
Velocity (ft/sec)	20	26	30	34	38	41	44	46	48	49	50

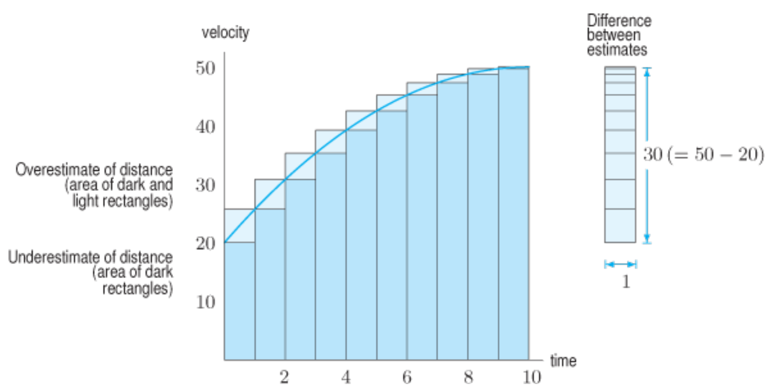
Use the table above to estimate the distance that the car traveled over the first ten minutes.

Visualizing Distance on the Velocity Graph

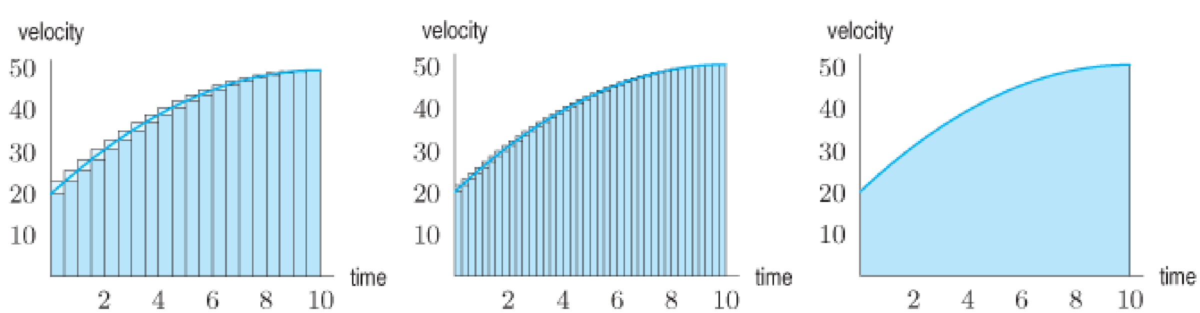
The following image shows the geometric representation of the calculations we performed for the case where velocity measurements were taken every 2 seconds.



Now take a look at the representation for the calculations we performed when velocity measurements were taken every second. Discuss.



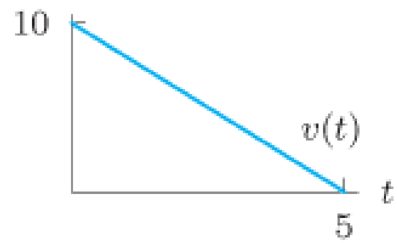
What do you expect will happen if we start to take measurements over shorter and shorter time intervals? What if we performed the same calculations if velocity is measured every 0.5 seconds? Every 0.25 seconds? What do you think will happen to the graph as $\Delta t \to 0+$?



> If velocity is positive, the total distance traveled is the area under the velocity curve.

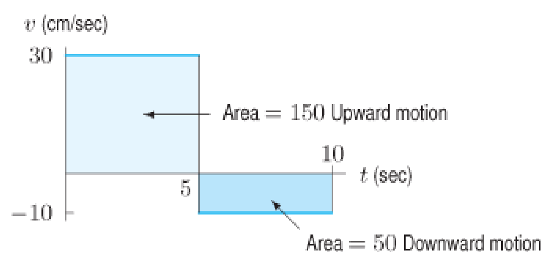
Examples

1. Use the graph of the velocity curve to calculate the distance traveled.



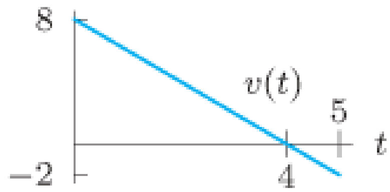
What Happens if the Velocity is Negative?

The following figure shows the velocity graph for a particle traveling at 30 cm/s eastward, and then instantaneously changing course and traveling at 10 cm/s westward.



In such a case, we count the areas under the time-axis as "negative area", and instead of interpreting the combined "areas" as the total distance traveled, we now interpret it as the total change in position.

2. Use the velocity graph below to find the total change in position.



Left and Right Sums

Let $v = f(t)$ denote a velocity function. We will write down semi-explicit formulas for the rectangular estimates we saw previously.

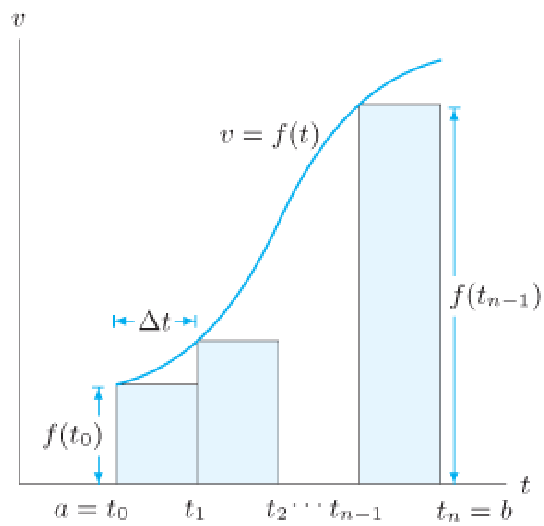
Suppose we want to calculate the total change in position from $t = a$ to $t = b$, and that we wish to use n rectangles to calculate the change in position. Then we take velocity measurements every

$$\Delta t = \frac{b-a}{n}$$

time units.

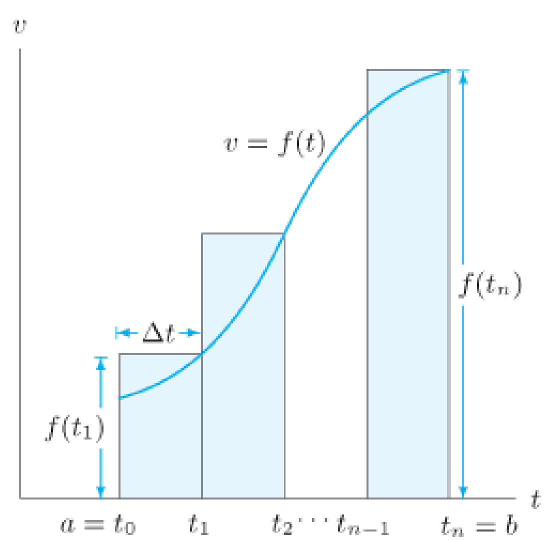
Left Sum: The left sum is given by

$$\text{Left Sum} = f(t_0)\Delta t + f(t_1)\Delta t + f(t_2)\Delta t + \cdots + f(t_{n-1})\Delta t.$$



Right Sum: The right sum is given by

$$\text{Right Sum} = f(t_1)\Delta t + f(t_2)\Delta t + f(t_3)\Delta t + \cdots + f(t_n)\Delta t.$$



Examples:

3. At time t, in seconds, your velocity v, in meters/second, is given by
$$v(t) = 1 + t^2 \text{ for } 0 \leq t \leq 6.$$
Use $\Delta t = 2$ to estimate the distance traveled during that time. Find the upper and lower estimates, and then average the two.

4. The velocity of a particle moving along the x-axis is given by $f(t) = 6 - 2t$ cm/s. Use a graph of $f(t)$ to calculate the exact change in position from $t = 0$ to $t = 4$.

5. A woman drives 10 miles, accelerating uniformly from rest to 60 mph. Graph her velocity vs. time. How long does it take for her to reach 60 mph?