Section 6.1: Antiderivatives Graphically and Numerically

As previously mentioned, if $F'(x) = f(x)$ for some continuous function $f(x)$, we call $F(x)$ an *antiderivative* of $f(x)$. The fundamental theorem of calculus relates the definite integral of functions over a closed interval to their antiderivatives by

$$\int_a^b f(x)\,dx = F(b) - F(a).$$

In this section, we are interested in trying to understand the general behavior of antiderivatives, and how we can discern that from the original function. Let us start with a simple question.

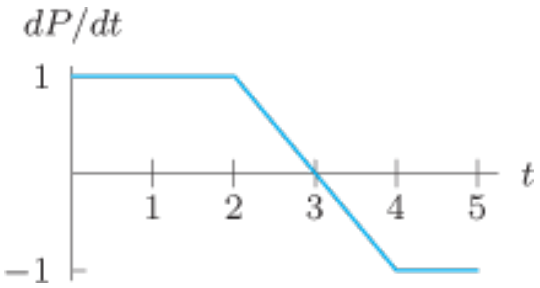
Question: What is an antiderivative of $f(x) = 2x$?

Is this antiderivative unique? If not, write a general expression for the *family of antiderivatives*.

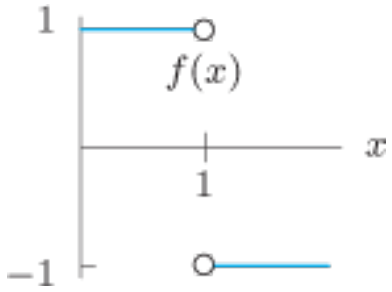
In the following problems, we will be asked to find values of antiderivatives and to sketch graphs of antiderivatives given a graph of the original function and some *initial condition*. The fundamental theorem can be useful in doing this.

Examples:

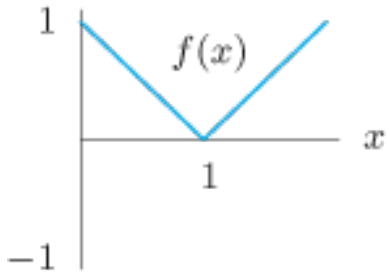
1. Use the figure below and the fact that $P = 0$ when $t = 0$ to find values of P when $t = 1$, 2, 3, 4, and 5.



2. Sketch two function F such that $F' = f$. In one case let $F(0) = 0$ and in the other let $F(0) = 1$.



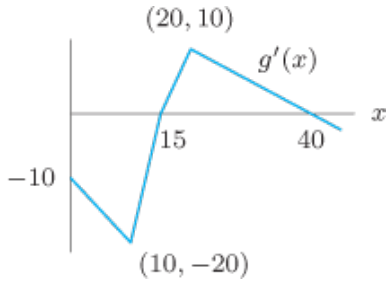
3. Sketch two functions F such that $F' = f$. In one case let $F(0) = 0$ and in the other let $F(0) = 1$.



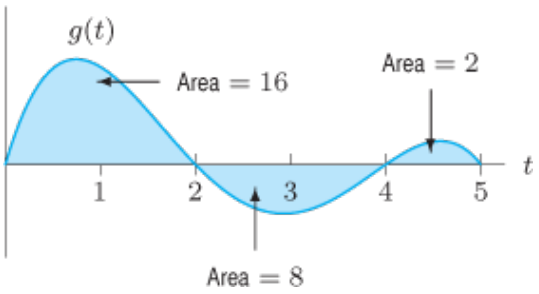
4. Estimate $f(x)$ for $x = 2$, 4, 6 using the values of $f'(x)$ and the fact that $f(0) = 50$.

x	0	2	4	6
$f'(x)$	17	15	10	2

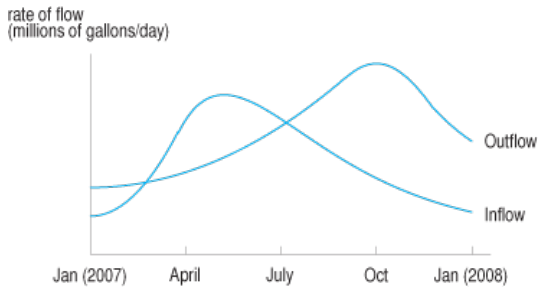
5. Using the graph of g' and the fact that $g(0) = 50$, sketch a graph of $g(x)$. Give the coordinates of all of the critical points and inflection points of g.



6. Using the figure below, sketch a graph of an antiderivative $G(t)$ of $g(t)$ satisfying $G(0) = 5$. Label each critical point of G with its coordinates.



7. The Quabbin reservoir in the western part of Massachusetts provides most of Boston's water. The graph below represents the flow of water in and out of the Quabbin reservoir throughout 2007.



(a) Sketch a graph of the quantity of water in the reservoir, as a function of time.

(b) When, in the course of 2007, was the quantity of water in the reservoir the largest? Smallest? Mark and label these points in your graph.

(c) When was the quantity of water increasing the most rapidly? Decreasing the most rapidly? Mark and label these points on your graph.