Section 6.3: Differential Equations and Motion

An equation of the form

$$\frac{dy}{dx} = f(x)$$

is called a *differential equation*. Finding the *general solutions* to the equation means finding the general antiderivative $y = F(x) + C$.

Examples:

1. Find the general solution to the differential equation.

   (a) $\frac{dy}{dt} = t^2$

   (b) $\frac{dy}{dt} = e^t$

   (c) $\frac{dy}{dx} = \frac{1}{x}$, where $x > 0$.

Initial Value Problems:

Finding the solution to an initial value problem means finding a solution to a differential equation whose curve passes through a specified point (x_0, y_0). That is, solve an equation of the form

$$\frac{dy}{dx} = f(x), \qquad\qquad f(x_0) = y_0.$$

Examples:

2. Find the solution to the initial value problem.

   (a) $\dfrac{dy}{dx} = x^5 + x^6$, $y(1) = 2$.

   (b) $\dfrac{dy}{dx} = \sin x$, $y(0) = 3$.

Equations of Motion:

It is a straightforward matter to use elementary differential equations to analyze the motion of an object falling freely under only the influence of gravity. An object falling under the influence of gravity has a constant acceleration, which we denote by g, where $g = 9.8$ m/s^2 or $g = 32$ ft/s^2.

Therefore, if v represents the upward velocity (positive velocity indicating upward motion), we have

$$\frac{dv}{dt} = -g.$$

Examples:

3. A rock is thrown downward with velocity 10 ft/s from a bridge 100 ft above the water. How fast is the rock going when it hits the water?

4. A water balloon is launched from the roof of a building at time $t = 0$ and has vertical velocity $v(t) = -32t + 40$ ft/s at time t seconds, with $v > 0$ corresponding to upward motion.

   (a) If the roof of the building is 30 ft above the ground, find an expression for the height of the water balloon at time t.

   (b) What is the average velocity of the balloon between $t = 1.5$ s and $t = 3$ s?

   (c) A 6-foot person is standing on the ground. How fast is the water balloon falling when it strikes them on the top of the head?