

Exam 1 Review Worksheet

MATH 122B · ~~Fall 2014~~

Instructor: Michael James Gilbert.

Answer the following questions to the best of your ability. You will be required to show all work on the exam.

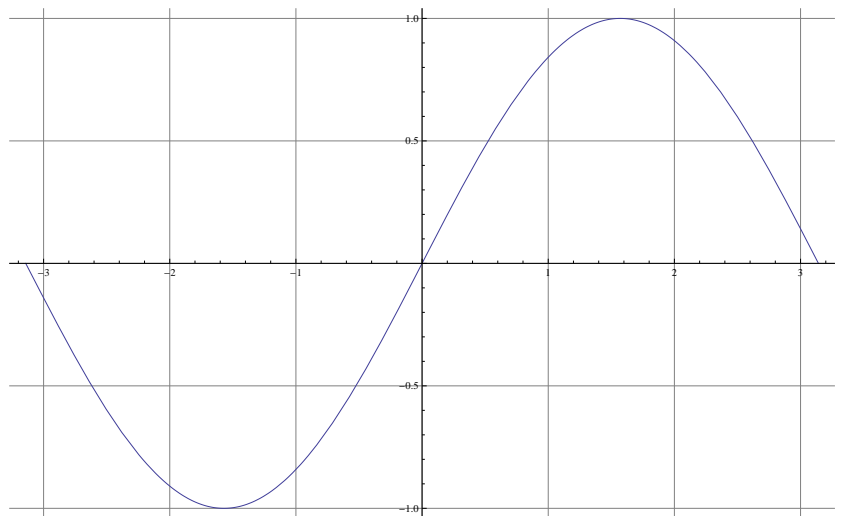
1. Find $\frac{d\varphi}{d\alpha}$ *using the definition of the derivative*

$$\varphi = \frac{1}{\sqrt{\alpha + 5}}$$

2. Find $\frac{dy}{dx}$ *using the definition of the derivative*

$$y = \frac{1}{(x+3)^2}$$

3. Below is shown the graph of $\beta(t) = \sin t$ for $-\pi \leq t \leq \pi$. On the same graph, sketch the line tangent to β at $t = 0$. Use this to estimate the value of $\beta'(0)$.



Using your knowledge of the behavior of $\beta(t) = \sin t$, what are the values of t for which $\beta'(t) = 0$ on the interval $-\pi \leq t \leq \pi$?

4. Is there a value of n which makes $y = x^n$ a solution to the equation $13x(dy/dx) = y$? If so, what value?

5. Given a power function $f(x) = ax^n$, with $f'(2) = 3$ and $f'(4) = 24$, find n and a .

6. Some antique furniture increased very rapidly in price over the past decade. For example, the price of a particular rocking chair is well approximated by

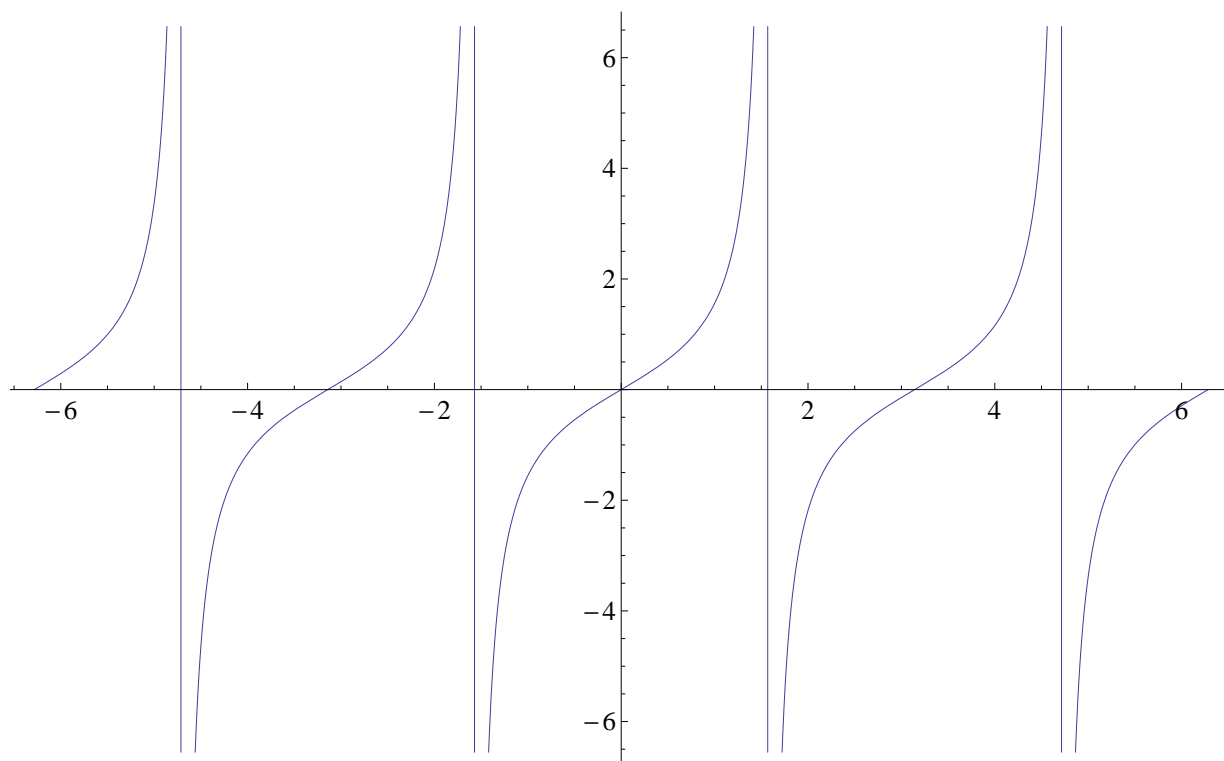
$$V = 75(1.35)^t,$$

where V is in dollars and t is in years since 2000. Find the rate, in dollars per year, at which the price is increasing at time t .

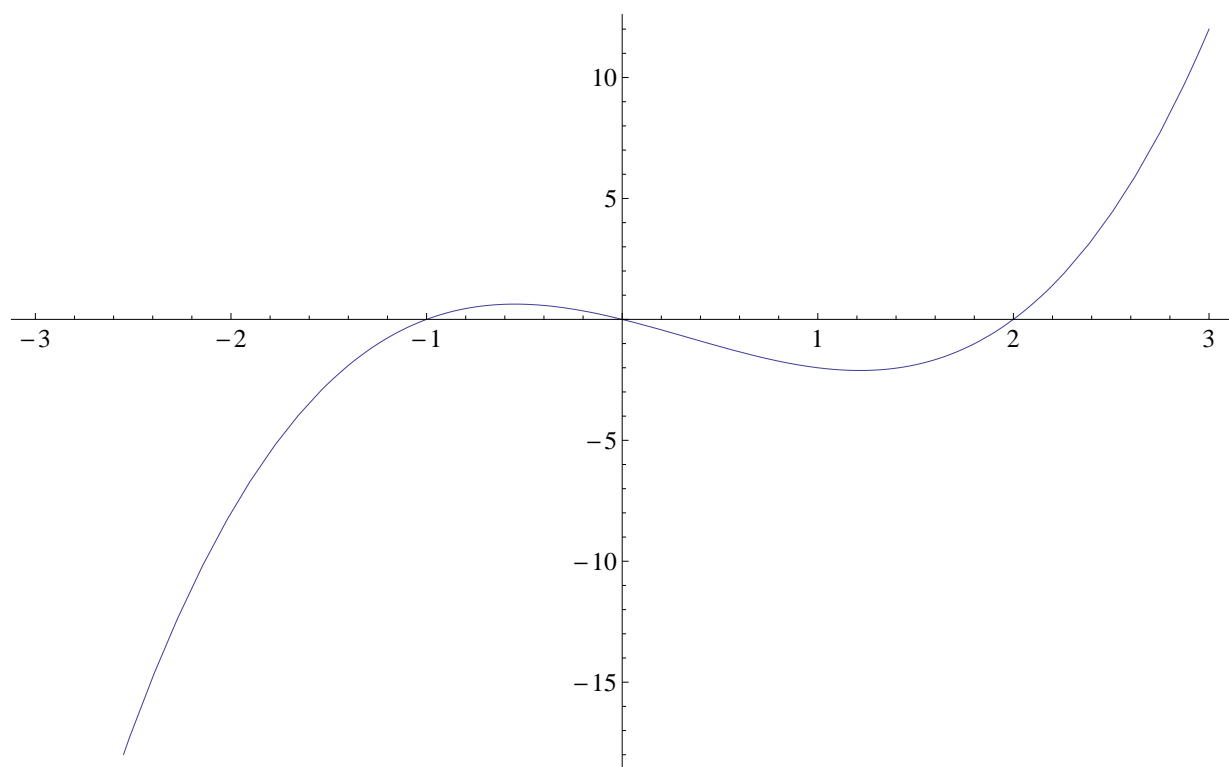
7. For what intervals is $f(x) = x^3 + 36x$ concave up?

8. For what intervals is $f(x) = x^3 + 2x^2 - 7x + 3$ concave down?

9. The graph of $y = \tan x$ is shown below. Use the figure, along with your knowledge of the way the derivative works, to sketch a graph of $\frac{dy}{dx}$ on the same axes.



10. The graph of the function $w = g(z)$ is shown below. Sketch a graph of $\frac{dw}{dz}$ on the same axes.



11. Let $f(3) = 6$, $g(3) = 12$, $f'(3) = \frac{1}{2}$, and $g'(3) = \frac{4}{3}$. Evaluate the following when $x = 3$.

$$\frac{d}{dx}(2f(x) + 3g(x)) - (g(x) - 4f'(3)).$$

12. Find the equation of the line tangent to the function $f(x) = \left(\frac{1}{2}\right)^x + x^2$ at $x = 0$.

13. If t is the number of years since 2009, the population, P , of Ukraine, in millions, can be approximated by the function

$$P = f(t) = 45.7e^{-0.0016t}.$$

Estimate $f(6)$ and $f'(6)$, giving units. What do these two numbers tell you about the population of Ukraine?

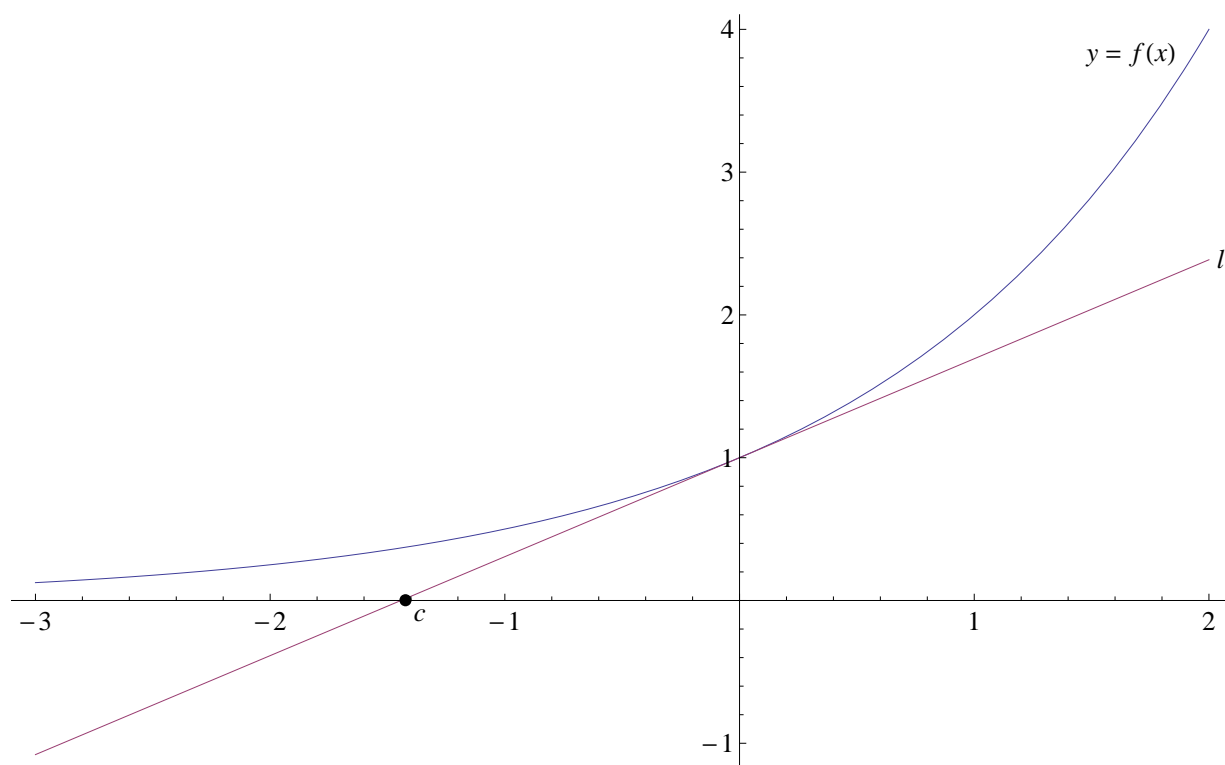
14. The quantity, q , of a certain skateboard sold depends on the selling price, p , in dollars, so we write $q = f(p)$. You are given that $f(140) = 15,000$ and $f'(140) = -100$.

(a) What do $f(140) = 15,000$ and $f'(140) = -100$ tell you about the sales of skateboards?

(b) What is the sign of $\left. \frac{dR}{dp} \right|_{p=140}$? If the skateboards are currently selling for \$140, what happens to revenue if the price is increased to \$141?



15. The following graph shows the function $f(x) = 2^x$ and the line tangent to f at the point $(0, 1)$.



Find the coordinates of the point labeled c .

16. A differentiable function f has the following properties:

- $f(100) = 140$,
- $f'(100) = 5$,
- $f''(x) > 0$ for $-80 < x < 120$.

(a) Find the equation of the tangent line to f at $x = 100$.

(b) Use your result from part (a) to estimate $f(105)$.

(c) Is your answer in part (b) an overestimate or an underestimate? *why?*

17. Draw a possible graph of $f(x)$ given the following information about its derivative.
- For $x < -2$, $f'(x) > 0$ and the derivative is increasing.
 - For $-2 < x < 1$, $f'(x) > 0$ and the derivative is decreasing.
 - At $x = 1$, $f'(x) = 0$.
 - For $x > 1$, $f'(x) < 0$ and the derivative is decreasing.

18. Find an equation for the tangent line of $f(x)$ at $x = 2$ if

$$f(x) = \frac{x^3}{2} - \frac{4}{3x}.$$

19. On what intervals is $f(x) = x^4 - 4x^3$ both increasing and concave up?

20. For what values of x is the graph of $y = x^5 - 5x$ both increasing and concave up.