Study Guide 2 · Math 122B

1. If $f(t) = 2t^3 - 4t^2 + 3t - 1$, find $f'(t)$ and $f''(t)$

2. If $f(x) = 13 - 8x + \sqrt{2}x^2$ and $f'(r) = 4$, find r.

3. Find the equations for the lines tangent to the graph of
$$xy + y^2 = 4$$
where $x = 3$.

4. Assume that y is a differentiable function of x and find dy/dx if $x^3 + y^3 - 4x^2y = 0$.

5. Assume that y is a differentiable function of x and find dy/dx if $\cos^2 y + \sin^2 y = y + 2$.

6. Find the slope of the curve $x^2 + 3y^2 = 7$ at $(2, -1)$.

7. Assume that y is a differentiable function of x and that $y + \sin y + x^2 = 9$. Find dy/dx at the point $x = 3, y = 0$.

8. Find derivatives for the following functions. Assume that a, b, c, and k are constants.

(a) $f(t) = e^{3t}$.

(b) $y = \frac{\sqrt{t}}{t^2+1}$.

(c) $f(x) = x^e$.

(d) $y = \sqrt{\theta}\left(\sqrt{\theta} + \frac{1}{\sqrt{\theta}}\right)$.

(e) $f(y) = \ln\left(\ln(2y^3)\right)$.

(f) $y = e^{-\pi} + \pi^{-e}$.

(g) $f(t) = \cos^2(3t + 5)$.

(h) $s(\theta) = \sin^2(3\theta - \pi)$.

(i) $p(\theta) = \frac{\sin(5-\theta)}{\theta^2}$.

(j) $q(\theta) = \sqrt{4\theta^2 - \sin^2(2\theta)}$.

(k) $g(t) = \arctan(3t - 4)$

(l) $m(n) = \sin(e^n)$.

(m) $g(t) = t\cos(\sqrt{t}e^t)$.

(n) $h(x) = xe^{\tan x}$.

(o) $f(t) = e^{-4kt}\sin t$.

(p) $g(\theta) = \sqrt{a^2 - \sin^2\theta}$.

(q) $f(s) = \frac{a^2 - x^2}{\sqrt{a^2+s^2}}$

(r) $r(t) = \ln\left(\sin\left(\frac{t}{k}\right)\right)$.

(s) $y = \frac{e^{2x}}{x^2+1}$.

(t) $z = \frac{e^{t^2}+t}{\sin(2t)}$.

(u) $g(y) = e^{2e^{y^3}}$.

(v) $h(t) = e^{kt}(\sin at + \cos bt)$.

(w) $r(\theta) = \sin\left((3\theta - \pi)^2\right)$.

(x) $f(y) = 4^y(2 - y^2)$.

(y) $h(x) = \left(\frac{1}{x} - \frac{1}{x^2}\right)(2x^3 + 4)$.

(z) $f(t) = (\sin(2t) - \cos(3t))^4$.

(zz) $f(\theta) = \theta^2 \sin\theta + 2\theta\cos\theta - 2\sin\theta$.

9. An electric current as a function of time, $I(t)$, is given (in units of amps) by the function

$$I(t) = \cos(\omega t) + \sqrt{3}\sin(\omega t).$$

Find all of the critical points of the function $I(t)$.

10. Find the critical points of the function $g(x) = xe^{-3x}$ and then classify them as local maxima or local minima or neither.

11. Find the critical points and inflection points of the function $f(x) = x^3 - 9x^2 + 24x + 5$.

12. Find the critical points and inflection points of the function $f(x) = x^5 + 15x^4 + 25$

13. Find the critical points and inflection points of the function $f(x) = 4xe^{3x}$.

14. Find all of the critical points and then use the first derivative test to determine local maxima and local minima for the function $f(x) = (x^2 - 4)^7$.

15. Find all of the critical points and then use the first derivative test to determine local maxima and local minima for the function $f(x) = \frac{x}{x^2+1}$.

16. Find the critical points of the function $h(x) = x + 1/x$ and classify them as local maxima or local mimima or neither.

17. (a) If a is a nonzero constant, find all critical points of
$$f(x) = \frac{a}{x^2} + x.$$

(b) Use the second-derivative test to show that if a is positive then the graph has a local minimum, and if a is negative then the graph has a local maximum.

18. Find $\lim_{h \to 0} \dfrac{e^{2(3+h)} - e^{2(3)}}{h}$ by recognizing the limit as the definition of $f'(a)$ for some function $f(x)$ and some value a.

19. Consider the function $f(t) = \dfrac{bt}{1 + at^2}$. Find values of a and b so that $f(t)$ has a critical point at $(4, 1)$.

20. Suppose that f has a continuous positive second derivative for all x. Which is larger, $f(1+\Delta x)$, or $f(1) + f'(1)\Delta x$? Explain.

21. Find the local linearization (the tangent line approximation) for $\sqrt{1+x}$ near $x = 0$.

22. Find the tangent line approximation to $1/x$ near $x = 1$.

23. Find the tangent line approximation to $\cos x$ at $x = \pi/4$.

24. For the following problems, find the local linearization of $f(x)$ near $x = 0$ and use this to approximate the value of a.

(a) $f(x) = (1+x)^r$, $a = (1.2)^{3/5}$.

(b) $f(x) = e^{kx}$, $a = e^{0.3}$.

(c) $f(x) = \sqrt{b^2 + x}$, $a = \sqrt{26}$.

The rest of the exercises you should all be able to do are the exercises we have done in class together.