Section 12.5: Functions of Three Variables

Representing a Function of Three Variables using a Family of Level Surfaces

Just as we could plot a family of *level curves* (a contour diagram) for a function $f(x, y)$ of two variables, we can "plot" a family of *level surfaces* for a function of three variables $w = f(x, y, z)$.

A *level surface*, or *level set* of a function of three variables, $f(x, y, z)$, is a surface of the form $f(x, y, z) = c$, where c is a constant. The function f can be represented by the family of level surfaces obtained by allowing c to vary.
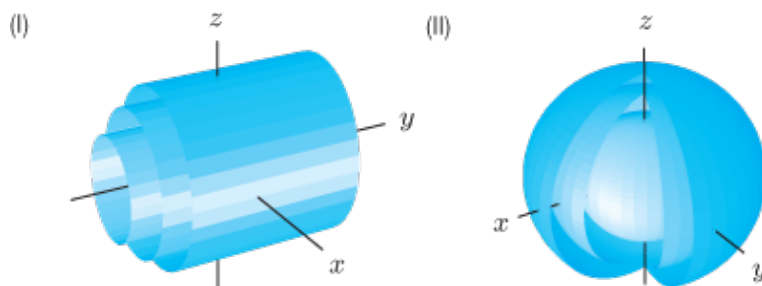

Temperature Example:

The temperature in °C, at a point (x, y, z) is given by $T(x, y, z) = x^2 + y^2 + z^2$. Pictured below is a family of level surfaces for this function. Discuss.
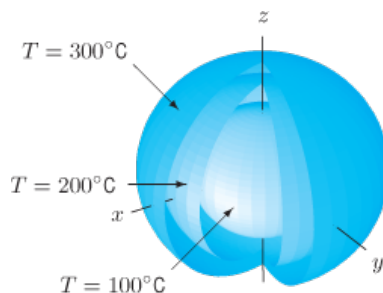
Examples:

1. Match the functions with the level surfaces depicted below:

(a) $f(x, y, z) = x^2 + y^2 + z^2$.

(b) $g(x, y, z) = x^2 + z^2$.

2. Find a formula for a function $f(x, y, z)$ whose level surfaces are spheres centered at (a, b, c).

3. Use the catalog of surfaces to identify the following surfaces:

   (a) $x^2 + y^2 - z = 0$

   (b) $x + y = 1$

   (c) $x^2 + y^2/4 + z^2 = 1$

4. Decide if the following level surfaces can be expressed as the graph of a function $f(x, y)$.

(a) $z - x^2 - 3y^2 = 0$.

(b) $x^2 + y^2 + z^2 - 1 = 0$.

How surfaces can represent functions of two variables and functions of three variables

Problem: Identify the surface given by $f(x, y) = e^{x^2} + y^2$ as a level surface of a three variable function $g(x, y, z)$.

A single surface that is the graph of a two-variable function $f(x, y)$ can be thought of as one member of the family of level surfaces representing the three variable function
$$g(x, y, z) = f(x, y) - z.$$
The graph of f is the level surface $g = 0$.

Examples:

5. Represent the bottom half of the ellipsoid $x^2 + y^2 + z^2/2 = 1$ as the graph of a two-variable function $f(x, y)$ and as a level surface of a three variable function $g(x, y, z) = c$.

6. Find a function $f(x, y, z)$ whose level surface $f = 1$ is the graph of the function $g(x, y) = x + 2y$.

7. Find a formula for a function $g(x, y, z)$ whose level surfaces are planes parallel to the plane $z = 2x + 3y - 5$.

8. Describe in words the level surfaces of $g(x, y, z) = e^{-(x^2+y^2+z^2)}$.