Section 13.2: Vectors in General

As you have most likely already deduced on your own, displacements aren't the only types of quantities that have both magnitude and direction, and which can be added together or scaled in the same way as displacement vectors. Any such quantity that satisfies those properties is called a *vector*, and can be represented in precisely the same way as a displacement vector.

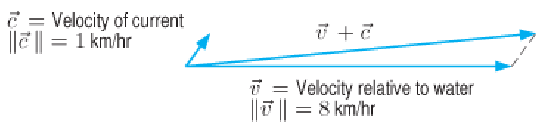
Question: What are the different types of vector quantities and scalar quantities that you can think of?

The **velocity vector** of a moving object is a vector whose magnitude is the speed of the object and whose direction is the direction of its motion.

The following example comes directly from the textbook, but helps illustrate some important concepts that will be useful.

Examples:

1. A riverboat is moving with velocity $\vec{v}$ and a speed of 8 km/hr relative to the water. In addition, the river has a current $\vec{c}$ and a speed of 1 km/hr. What is the physical significance of the vector $\vec{v} + \vec{c}$?



The following examples are not covered in detail in the textbook, but are indicative of the types of problems you might see from this section.

2. An airplane heads northeast at an airspeed of 700 km/hr, but there is a wind blowing from the west at 60 km/hr. In what direction does the plane end up flying? What is its speed relative to the ground?

3. An airplane is flying at an airspeed of 500 km/hr. A wind is blowing from southeast at 50 km/hr. In what direction should the plane head to end up going due east? What is the airplane's speed relative to the ground?

4. A man wishes to row the shortest possible distance from north to south across a river that is flowing at 4 km/hr from the east. He can row at 5 km/hr.

   (a) In which direction should he steer?

   (b) If there is a wind of 10 km/hr from the southwest, in which direction should he steer to try and go directly across the river? What happens?

5. An object P is pulled by a force $\vec{F}_1$ of magnitude 15 lb at an angle of 20 degrees north of east. In what direction must a force $\vec{F}_2$ of magnitude 20 lb pull to ensure that P moves due east?

Vectors in n Dimensions

Recall that we can write an arbitrary vector in 3-space, $\vec{v} = v_1\vec{i} + v_2\vec{j} + v_3\vec{k}$ as $\vec{v} = \langle v_1, v_2, v_3 \rangle$. Using this notation, we can easily conceive of a vector in n dimensions as a string of n numbers. We might write something like this as

$$\vec{c} = \langle c_1, c_2, \ldots, c_n \rangle.$$

Addition and scalar multiplication are then defined component by component as usual:

$$\vec{v} + \vec{w} = \langle v_1, v_2, \ldots, v_n \rangle + \langle w_1, w_2, \ldots, w_n \rangle = \langle v_1 + w_1, v_2 + w_2, \ldots, v_n + w_n \rangle$$

and

$$\lambda\vec{v} = \lambda\langle v_1, v_2, \ldots, v_n \rangle = \langle \lambda v_1, \lambda v_2, \ldots, \lambda v_n \rangle$$