

Section 14.1: The Partial Derivative

A *partial derivative* of a two-variable function $f(x, y)$ is nothing more than a derivative with respect to one of the variables while keeping the other variable held constant.

Partial Derivatives of f With Respect to x and y

For all points at which the limit exists, we define the *partial derivatives at the point* (a, b) by

$$f_x(a, b) = \lim_{h \rightarrow 0} \frac{f(a + h, b) - f(a, b)}{h},$$

and

$$f_y(a, b) = \lim_{h \rightarrow 0} \frac{f(a, b + h) - f(a, b)}{h}.$$

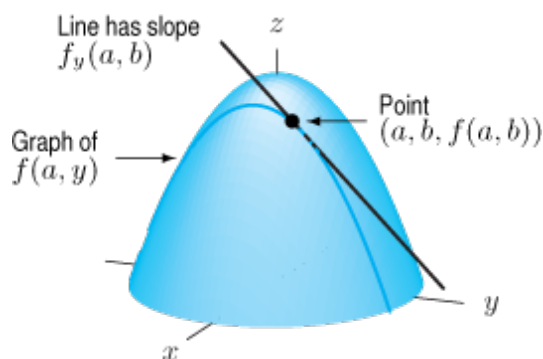
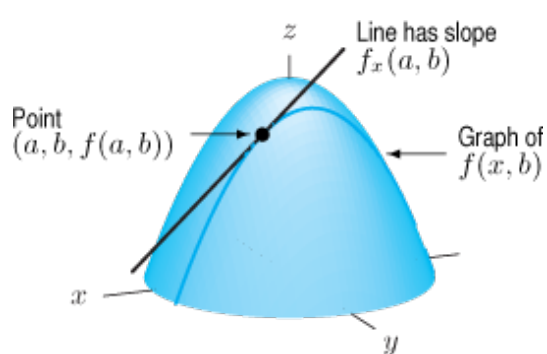
If we let a and b vary, we have the *partial derivative functions* $f_x(x, y)$ and $f_y(x, y)$.

Alternative Notation for Partial Derivatives

If $z = f(x, y)$, we can write

$$f_x(x, y) = \frac{\partial z}{\partial x} \text{ and } f_y(x, y) = \frac{\partial z}{\partial y}$$

$$f_x(a, b) = \left. \frac{\partial z}{\partial x} \right|_{(a, b)} \text{ and } f_y(a, b) = \left. \frac{\partial z}{\partial y} \right|_{(a, b)}$$



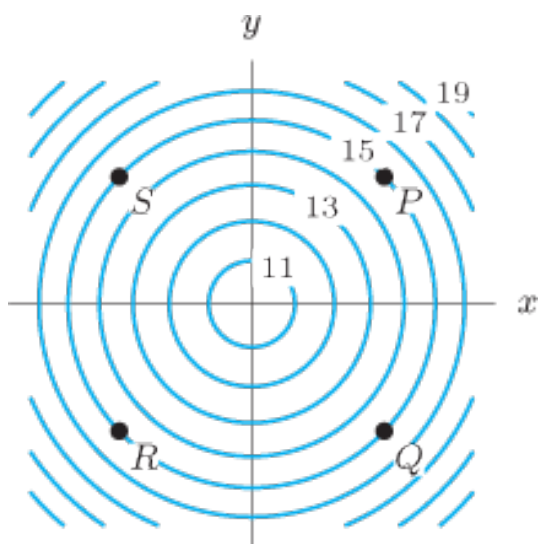
Examples:

1. Given the following table of values for $z = f(x, y)$, estimate $f_x(3, 2)$ and $f_y(3, 2)$. Assume that f is differentiable.

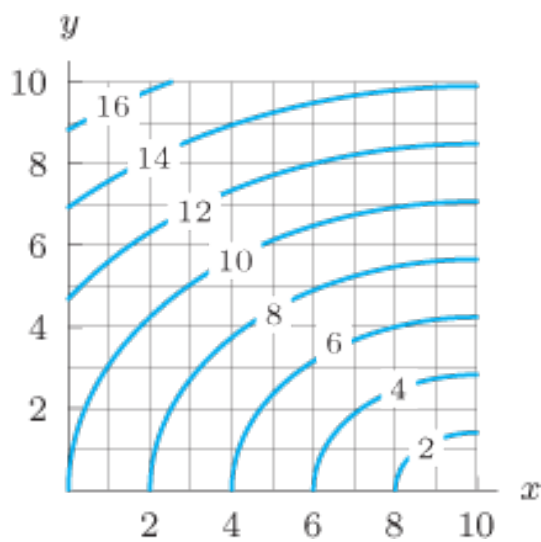
$x \backslash y$	0	2	5
1	1	2	4
3	-1	1	2
6	-3	0	0

2. The price P in dollars to purchase a used car is a function of its original cost, C , in dollars, and its age, A , in years.
- What are the units of $\partial P / \partial A$?
 - What is the sign of $\partial P / \partial A$ and why?
 - What are the units of $\partial P / \partial C$?
 - What is the sign of $\partial P / \partial C$ and why?

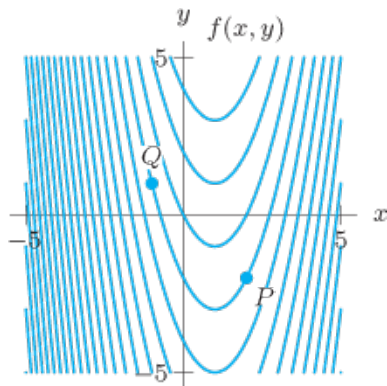
3. Determine the sign of f_x and f_y at the point using the contour diagram of f in the figure below.



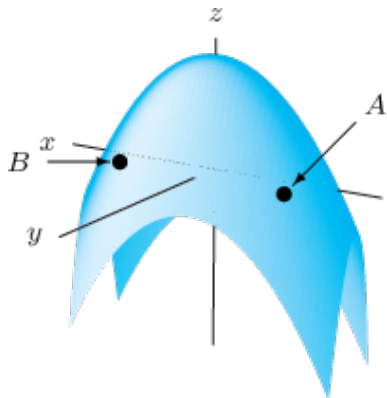
- (a) P
- (b) Q
- (c) R
- (d) S
4. Approximate $f_x(3, 5)$ using the contour diagram of $f(x, y)$ shown in the figure below.



5. The figure below shows the contours of $f(x, y)$ with values of f on the contours omitted. If $f_x(P) > 0$, find the signs of $f_y(P)$, $f_x(Q)$, and $f_y(Q)$.



6. The surface $z = f(x, y)$ is shown in the figure below. The points A and B are in the xy -plane.



- (a) What is the sign of
- (i) $f_x(A)$
 - (ii) $f_y(B)$
- (b) The point P moves in the xy -plane along a straight line from A to B . How does the sign of $f_y(P)$ change?