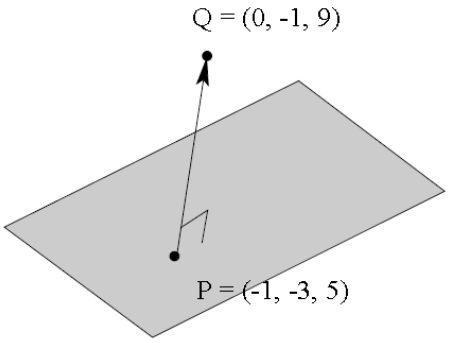
Team Homework 3

1. Find the angle between the planes $2x - 3y + z = 1$ and $-x - 3y + 2z = 5$

2. Find an equation for a plane containing the points $P = (1, 2, 3)$ and $Q = (3, 2, 1)$ that is perpendicular to the plane $4x - y + 2z = 7$.

3. The picture below shows a plane that contains the point $P = (-1, -3, 5)$. The plane is perpendicular to the vector $\vec{PQ}$, where $Q = (0, -1, 9)$.



   (a) Let $R = (1, 0, 3)$. Does the vector $\vec{PR}$ lie on the plane shown? If not, what is the angle between $\vec{PR}$ and the plane?

   (b) Let $S = (-2, 1, 2)$. Does the vector $\vec{PS}$ lie on the plane shown? If not, what is the angle between $\vec{PS}$ and the plane?

   (c) Suppose we are given a point $T = (x, y, z)$. Determine a general method for determining whether $\vec{PT}$ lies on the plane shown.

   (d) The plane in the picture divides space into two half-spaces, one containing Q and one not containing Q. Suppose that $T = (x, y, z)$ is a point that is not on the plane. How can we decide which half-space T is in?

4. Let $A = (-1, 3, 0)$, $B = (3, 2, 4)$, and $C = (1, -1, 5)$.

   (a) Find an equation for the plane that passes through these three points.

   (b) Find the area of the triangle determined by these three points.

5. (a) Find a function $f(x, y, z)$ whose level surface $f = 1$ is the graph of the function $g(x, y) = x + 2y$.

   (b) Find a function $f(x, y, z)$ whose level surface $f = -2$ is the graph of the function $g(x, y) = 2x^2 y e^{-(x^2+3y^3)}$.

   (c) If $g(x, y)$ is a two-variable function and k is any constant, describe a general method for finding a function $f(x, y, z)$ whose level surface $f = k$ is the graph of the function $g(x, y)$.