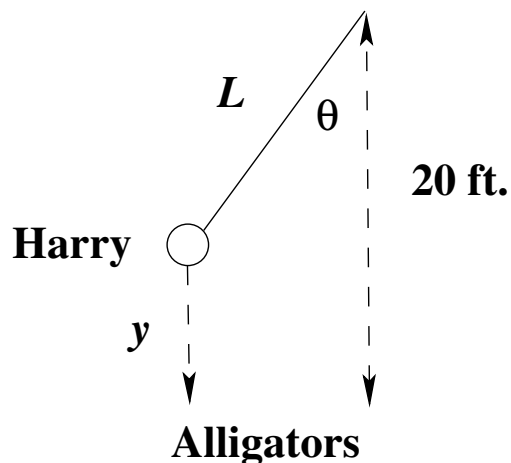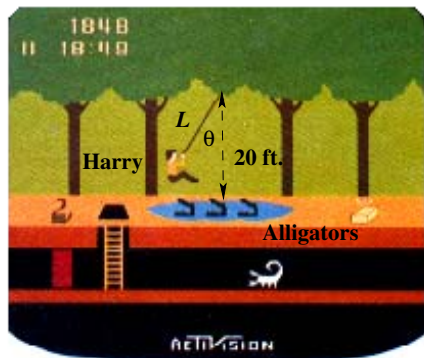# Team Homework 5

1. In the Atari game *Pitfall!*, Pitfall Harry searches for treasure in a jungle. At certain points, Harry has to grab a vine hanging from a tree and swing over a pond full of alligators. (See the figure on the left. A more "abstract" version of the figure is shown on the right.)

The vine has a length L (in feet) and swings from a point 20 feet over the pond. If t is the time in seconds from the moment Harry grabs the vine, the angle θ between the vine and the normal to the pond is given by

$$\theta(t) = \frac{\pi}{4}\cos(2t).$$

In addition, the vine is slightly elastic, and the vine stretches out at a rate of $dL/dt = 1/L$ at any given time while Harry is swinging.

(a) Let y be the distance from Harry to the pond (and the alligators). Write y as a function of L and θ.

(b) When $t = 0.4$ seconds, $L = 15$ feet. At what rate is y changing (with respect to time) at this time? Give a decimal approximation.

2. Define $g(x, y, z) = z^2 \sin(y - xz) + 2y$. Answer the following questions.

(a) Compute the gradient of g at the point $P = (1, 1, 1)$.

(b) The point P defined above is on the level surface $g = C$. What is the value of C?

(c) Find an equation of the tangent plane to the level surface $g = C$ at the point P.

(d) Suppose we want to travel from the point P to the level surface $g = C + 1$. **Estimate** how far we have to go if we go in the direction that gets us there the fastest?

3. Let $z = f(t)g(t)$. Use the chain rule applied to $h(x, y) = f(x)g(y)$ to show that

$$\frac{dz}{dt} = f'(t)g(t) + f(t)g'(t).$$