Team Homework 8

1. The right circular cone shown in the figure below has radius 1 foot and lateral height 2 feet. In each of the following problems, the cone has non-uniform density. Use the information given to determine the mass of the cone in each case.

   (a) The density of the cone at any point is proportional to the distance from the point to the central axis of the cone (the axis from the apex to the center of the base). The density of the cone at the circumference of the base is 4 pounds per cubic foot.

   (b) The density of the cone at any point is proportional to the distance from that point to the apex of the cone. The density of the cone at the circumference is 4 pounds per cubic foot.

2. Evaluate each of the integrals.

   (a) $\displaystyle\int_0^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_0^{\sqrt{4-x^2-y^2}} z\sqrt{x^2+y^2}\,dz\,dy\,dx.$

   (b) $\displaystyle\int_0^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_0^{\sqrt{4-x^2-y^2}} \sqrt{x^2+y^2}\,dz\,dy\,dx.$

3. Below is a picture of a delicious bundt cake:

   The shape of this bundt cake can be modeled by sketching the parabola $z = -y^2 + 6y - 5$ in the yz-plane (all lengths are in inches), and then revolving the bounded region between this parabola and the y-axis around the z-axis to create a three-dimensional solid. However, when the cake was baked, the ingredients did not settle uniformly. Let $\rho(x, y, z)$ represent the density of the cake (in pounds per cubic inch) at the point (x, y, z). Set up an integral, including limits of integration, for the total weight of the cake.