

Exam 1 Study Guide

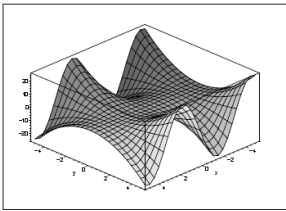
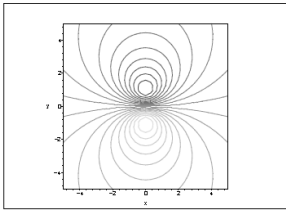
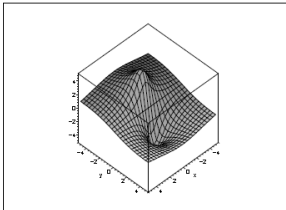
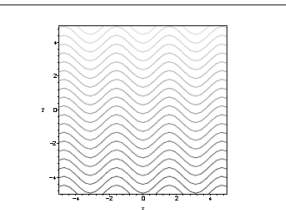
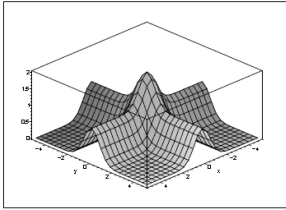
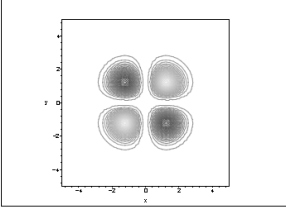
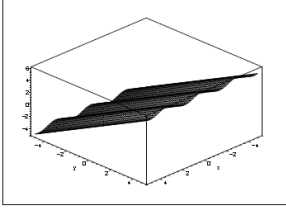
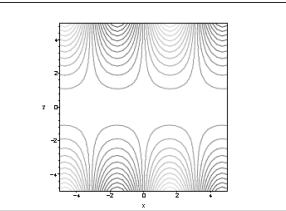
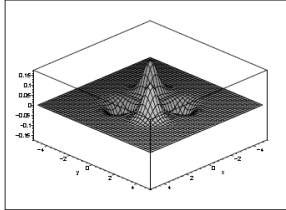
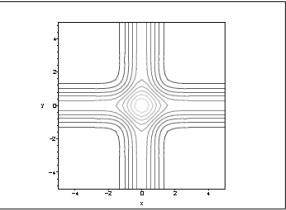
Math 223

1. Consider the points $A = (1, -3, 2)$, $B = (3, -1, -1)$, $C = (-4, -9, -8)$.

(a) Find an equation of the plane containing the points A , B , and C .

(b) Find the area of the triangle formed by A , B , and C .

2. Match the following functions, graphs, and level curves.

(a) $f(x, y) = x^3 y^3 e^{-x^2 - y^2}$	(I) 	(A) 
(b) $f(x, y) = \frac{-10y}{x^2 + y^2 + 1}$	(II) 	(B) 
(c) $f(x, y) = y^2 \sin x$	(III) 	(C) 
(d) $f(x, y) = e^{-x^2} + e^{-y^2}$	(IV) 	(D) 
(e) $f(x, y) = \cos^2 x + y$	(V) 	(E) 

3. Find or estimate, depending on the type of data provided, partial derivatives f_x and f_y at the point $P = (3, 2)$ for the following four functions.

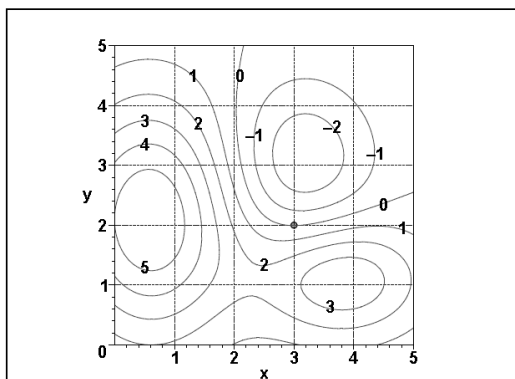
- (a) The function f_1 is given by the formula

$$f_1(x, y) = x^2 \cos(xy)$$

- (b) Values for the function f_2 are given by the table:

$f_2(x, y)$	$x = 2.8$	$x = 3.0$	$x = 3.2$	$x = 3.4$
$y = 2.1$	1.20	1.35	1.50	1.64
$y = 2.0$	1.26	1.41	1.55	1.70
$y = 1.9$	1.31	1.46	1.61	1.75
$y = 1.8$	1.37	1.52	1.66	1.81

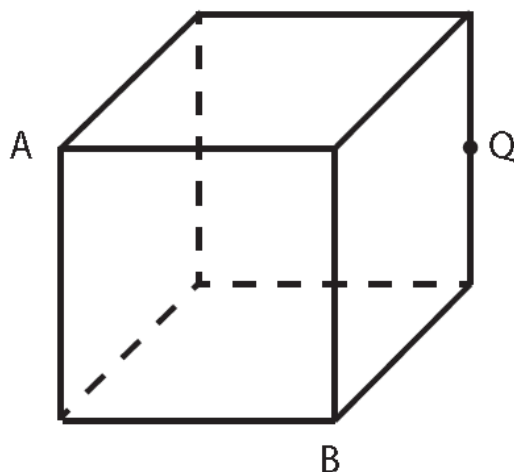
- (c) The function f_3 is given by its level curves



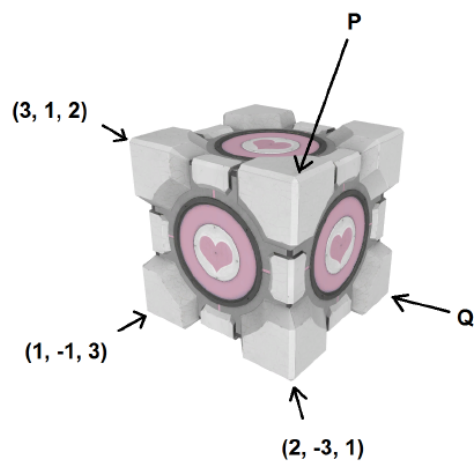
4. This problem consists of three unrelated questions on vector algebra and geometry.
- Two vectors $\vec{v}$ and $\vec{w}$ are of the form $\vec{v} = a(\hat{i} + \hat{j})$, and $\vec{w} = b\hat{i} + \hat{j}$, where a and b are scalars. If $\vec{v} + \vec{w} + 10\hat{i} - \hat{j} = \vec{0}$, find the scalars a and b .
 - A constant force $\vec{F} = -\hat{i} + 2\hat{j} - 5\hat{k}$ acts on a particle as it moves from the point $(2, 4, 1)$ to $(-2, 1, 3)$. Find the work done by the force.
 - Find the area of the parallelogram spanned by the vectors $\hat{i} - \hat{k}$ and $-2\hat{i} + 2\hat{j}$.
5. Let $A = (2, -5, 6)$, $B = (3, -5, 7)$, and $C = (3, -6, 8)$.
- Compute $\vec{AB} \times \vec{AC}$ and $\vec{AB} \cdot \vec{AC}$.
 - Find the angle between $\vec{AB}$ and $\vec{AC}$.
 - Find an equation for the plane containing A , B , and C .
6. Show that the following three relationships hold for any vectors $\vec{a}$, $\vec{b}$, and $\vec{c}$.
- $||\vec{a} \times \vec{b}||^2 = ||\vec{a}||^2 ||\vec{b}||^2 - (\vec{a} \cdot \vec{b})^2$.
 - If $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, then $\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a}$.
 - $\vec{a} \cdot (\vec{b} \times \vec{c}) = (\vec{a} \times \vec{b}) \cdot \vec{c}$.
7. Consider the following three planes:
- Plane A: $x - y = 2z + 1$ Plane B: $x - z = \frac{3 - y}{2}$ Plane C: $x = y + 2z$.

Which of the above planes are parallel to each other?

8. Consider a cube with side lengths of 2: see the picture below. Let A and B be the vertices indicated in the picture. Let Q be the point which is in the middle of an edge on the box (equal distance from the two endpoints of the edge) as indicated in the picture.



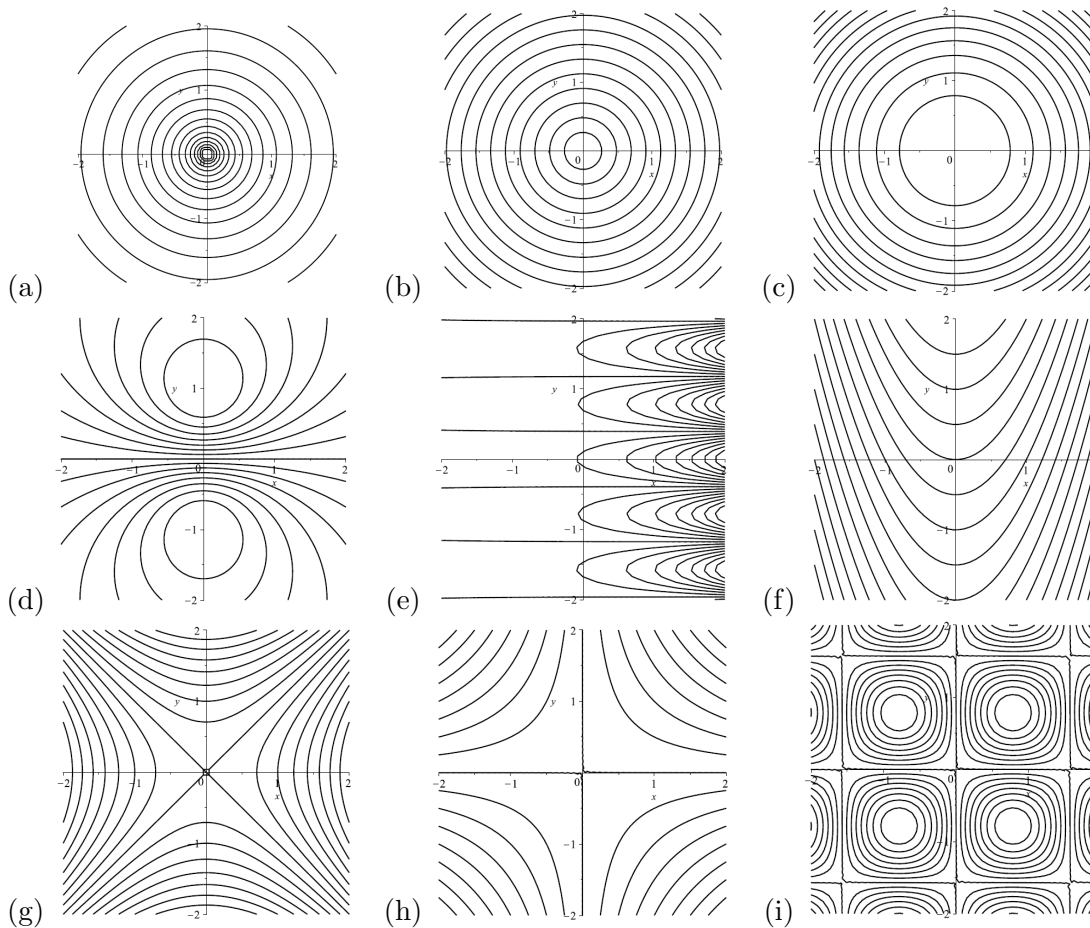
- (a) What is the angle between the line segment QA and the line segment QB ? (You can leave your answer as the inverse cosine of a real number.)
- (b) What is the area of the triangle formed by A , B , and Q ?
9. In a gesture of goodwill (or perhaps just wanting to move a test along), GLaDOS offers Chell a Companion Cube, a cubical box that falls out of a hatch in the ceiling. At one point in the cube's descent, three of the vertices of the box are at $(3, 1, 2)$, $(1, -1, 3)$, and $(2, -3, 1)$, as shown below. (*Note:* Even though the box itself is not a perfect cube, the vertices do form a cube.)



- (a) What are the coordinates of the vertex P , shown above?
- (b) What are the coordinates of vertex Q , shown above?

10. Find an equation of the plane that passes through the point $(-1, 2, 1)$ and is perpendicular to the line of intersection of the planes $x + y - z = 2$ and $2x - y + 3z = 1$.

11. Consider the level curves shown below, and answer the following questions.



(I) Which picture represents the level curves of $f(x, y) = \sqrt{x^2 + y^2}$?

(II) Which picture represents the level curves of $f(x, y) = x^2 - y^2$?

(III) The picture (i) is a contour diagram of a function among the following choices. Circle the correct function.

(A) $\sin x \sin y$ (B) $\sin x + \sin y$ (C) $\sin(xy)$ (D) $\sin(x + y)$

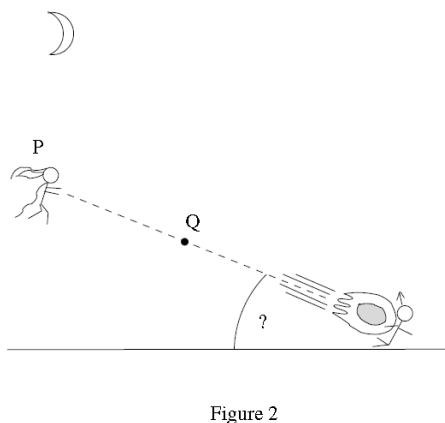
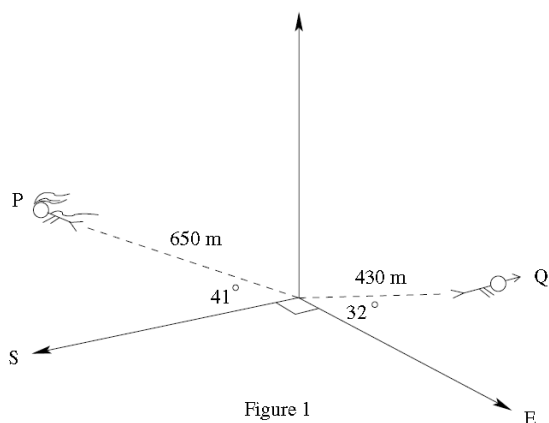
(E) $\frac{1}{x^2 + y^2 + 1}$ (F) $\frac{x + y}{x^2 + y^2 + 1}$ (G) $\frac{xy}{x^2 + y^2 + 1}$ (H) $x^2 + y^2 + 1$

12. Let $\vec{A} = 2\hat{i} + 3\hat{j} + \hat{k}$. Let P be the plane $x + y + z = 7$. Write the vector $\vec{A}$ as a sum

$$\vec{A} = \vec{B} + \vec{C}$$

where $\vec{B}$ is perpendicular to the plane P and $\vec{C}$ is parallel to the plane P .

13. The superheroine Differentia is battling her arch-nemesis Vek-Tor for control of the city of Mathtropolis. Hoping to destroy Vek-Tor from the sky, Differentia takes off southward from the center of the city at an angle of 41° with the ground and flies a distance of 650 meters to a point P . Hoping to avoid her, Vek-Tor takes off eastward from the same spot at an angle of 32° with the ground and flies a distance of 430 meters to a point Q . (See Figure 1.)



- (a) Once Differentia and Vek-Tor have reached points P and Q , respectively, how far apart are they?
- (b) Differentia aims an enormous fireball at Vek-Tor and blasts him with such great force that the fireball drives him backwards until he hits the ground. (See Figure 2.) At what angle does Vek-Tor crash into the ground?

14. Choose the items from the following list that best describes the given object.

- | | | | |
|----------------|-----------------|--------------|---------------|
| (A) Line | (B) Circle | (C) Parabola | (D) Hyperbola |
| (E) Plane | (F) Cone | (G) Sphere | (H) Cylinder |
| (I) Paraboloid | (J) Hyperboloid | (K) Saddle | |

(a) The graph of $f(x, y) = x^2 + y^2 - 4$

(b) The $g = 0$ level set of $g(x, y) = 3x + 5y + 1$

(c) The $p = 0.4$ level set of $p(x, y, z) = e^{-(x^2+y^2+z^2)}$

(d) The $h = 7$ level set of $h(x, y, z) = x^2 - y^2 - z$.

15. Suppose that the wind is blowing from the northwest at a speed of 50 km/hr. If a pilot is flying a plane with an airspeed of 250 km/hr, what direction must he travel (relative to the air) in order to end up going a true course due east? What is his groundspeed?

16. A tow truck drags a stalled car. The chain makes an angle of 30° with the road and the tension in the chain is 1500 Newtons. Find the work done by the tow truck in dragging the car for 1 km along the road.