30. Calculate the following (note, the calculator may not always give the complete answer!):

(a) $625^{1/4}$

(b) $(-625)^{1/4}$

(c) $125^{1/3}$

(d) $(-125)^{1/3}$

---

31. Simplify the following expressions, (assume all variables are nonnegative real numbers)

(a) $\sqrt[3]{625x^4}$

(b) $3\sqrt{48} - 5\sqrt{27}$

---

32. Rewrite $\sqrt[25]{x^7}$ as a fractional power.

33.

(a) Write 2,300,000,000,000 in scientific notation.

(b) Write 0.000000000000023 in scientific notation.

(c) Write 6.5×10^{-9} in standard notation.

(d) How many zeros does 6.5×10^{90} have?

34. By completing this question you should have a good idea of the differences of Linear Functions and Exponential Functions.

(a) What does a basic linear function look like?

$y = mx + b$

(b) What does a basic exponential function look like?

$y = Ca^x$

(c) If my function $f(t) = 150 + 50t$ models $f(t)$ the turtle population at t years. What does 150 represent? What does 50 represent? (in the context of the problem)

150 represents the initial population of turtles. 50 represents the increase in the turtle population each year.

(d) If my function $f(t) = 150(1.29)^t$ models $f(t)$ the turtle population at t years. What does 150 represent? What does 1.29 represent? (in the context of the problem)

150 represents the initial turtle population.
1.29 represents the annual growth factor.

(e) If you were given a table of x and y values, how could you tell if the table represented a linear function? The A.R.C. remains constant.

(f) If you were given a table of x and y values, how could you tell if the table represented an exponential function? The ratio of y-values is constant for equally spaced x-values.

(g) Fill in the blank: A linear function represents a quantity to which a constant amount is added for each unit increase in the input.

(h) Fill in the blank: An exponential function represents a quantity that is multiplied by a constant factor for each unit increase in the input.

(i) What does the average rate of change do within a linear function?

It stays constant

(j) What does the average rate of change do within an exponential function?

It increases exponentially for growth, and decays exponentially for decay.

(k) If you were given a linear function $y = mx + b$. Can m be negative? If so, what does this tell us? Yes. If the slope is negative, this means that the function values decrease for increasing x-values.

(l) If you were given a linear function $y = mx + b$. Can m be positive? If so, what does this tell us? Yes. It means the function values increase for increasing values of x.

(m) If you were given a linear function $y = mx + b$. Can m be 1? If so, what does this tell us?

Yes. It tells us that the slope is 1. i.e. for each unit increase in x, y increases by 1 unit.

(n) If you were given an exponential function $y = Ca^x$ can a be negative? If so, what does this tell us? No

(o) If you were given an exponential function $y = Ca^x$ can a be positive? If so, what does this tell us? (Hint: you need to separate into two cases, $a > 1$ and $0 < a < 1$)

Yes. If $a>1$, then the function represents exponential growth. If $0<a<1$, the function represents exponential decay.

(p) If you were given an exponential function $y = Ca^x$ can a be 1? If so, what does this tell us?

Technically yes, but the function fails to be exponential in this case, and becomes a constant function.

(q) If you have an exponential growth function, $y = Ca^x$, what must be the restriction?

$a>1$

(r) If you have an exponential decay function, $y = Ca^x$, what must be the restriction?

$0<a<1$

(s) If you have an exponential growth function, $y = Ca^x$, what is true when $x \to -\infty$? And what does this say graphically? As $x \to -\infty$, the function values approach zero. This takes the form of a horizontal asymptote in the graph.

(t) If you have an exponential decay function, $y = Ca^x$, what is true when $x \to \infty$? And what does this say graphically? As $x \to +\infty$, the function values approach zero. This takes the form of a horizontal asymptote in the graph.

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35. You are studying the population of red-eared slider turtles. You have come up with the following equation to represent the turtle population t years after you initially started studying their population.

$$p(t) = 120(1.4)^t.$$

(a) What does 120 mean in the context of the problem?

This was the population of turtles when you initially started studying them.

(b) What does 1.4 mean in the context of the problem?

This is the growth factor. Every year, the population of turtles grows by a factor of 1.4.

(c) What is the growth/decay rate and what does this mean in the context of the problem?

Growth rate = 40% $(1.4 - 1 = 0.4)$

This means that the population of red-eared slider turtles increases by 40% per year, according to your model.

36. Answer the following questions

(a) A population of something grows exponentially, in 2002 the population was 120 million, in 2010 the population was 340 million. Write the function, $P(t)$, population in millions where t is years since 2000 that represents this data.

$\cancel{P(0)=?}$ $\quad P(2)=120, \; P(10)=340$

$$\Rightarrow \left.\begin{array}{l}120 = Ca^2,\\ 340 = Ca^{10}\end{array}\right\} \Rightarrow \left\{\frac{340}{120} = \frac{Ca^{10}}{Ca^2}\right\} \Rightarrow \frac{17}{6} = a^8$$

$$\Rightarrow a = \left(\frac{17}{6}\right)^{1/8} \Rightarrow 120 = C\cdot\left(\tfrac{17}{6}\right)^{2/8} \Rightarrow C = \frac{120}{(17/6)^{1/4}} \approx 92.4926$$

$$\Rightarrow \boxed{P(t) = \frac{120}{(17/6)^{1/4}}\left(\tfrac{17}{6}\right)^{t/8}} \text{ (Exact) or}$$

$$\boxed{P(t) = 92.4926\,(1.1390)^t} \text{ (Rounded to 4 decimals)}$$

(b) A population has a doubling time of 10 years. If the population started at 5, write the function $P(y)$ where y is one year. Write the function $P(d)$ where d is in decades. Write the function $P(m)$ where m is in months. (Do exact and round growth (or decay) factor to 4 decimals.)

	Exact	Rounded
Years	$P(y) = 5\cdot 2^{y/10}$	$P(y) = 5\cdot(1.0718)^y$
Decades	$P(d) = 5\cdot 2^d$	$P(d) = 5\cdot 2^d$
Months	$P(m) = 5\cdot 2^{m/120}$	$P(m) = 5\,(1.0058)^m$

(c) A substance decays exponentially. At the 5th hour there is 500 mg at the 10th hour there is 300 mg. Write the function $S(h)$, mg of the substance, where h is in hours.

$$S(5) = 500, \quad S(10) = 300$$

$$\Rightarrow 500 = Ca^5, \quad 300 = Ca^{10}$$

$$\Rightarrow \cancel{\frac{500}{300}} \ \frac{300}{500} = \frac{\cancel{C}a^{10}}{\cancel{C}a^5} \Rightarrow a^5 = \frac{3}{5} \Rightarrow a = \left(\frac{3}{5}\right)^{1/5}$$

$$\Rightarrow 500 = C\left(\left(\frac{3}{5}\right)^{1/5}\right)^5 = \frac{3}{5} \cdot C \Rightarrow C = \frac{5}{3} \cdot 500 = \frac{2500}{3}$$

$$\Rightarrow \boxed{S(h) = \frac{2500}{3}\left(\frac{3}{5}\right)^{h/5}}$$

(d) Something is decaying exponentially. At the start of the study there are 15 units, 10 hours after there are 2.3 units. Write the function $U(h)$, number of units, for h hours.

$$U(0) = 15 \Rightarrow C = 15$$

$$U(10) = 2.3 \Rightarrow 2.3 = 15a^{10} \Rightarrow a^{10} = \frac{2.3}{15} = \frac{23}{150}$$

$$\Rightarrow a = \left(\frac{23}{150}\right)^{1/10}$$

$$\Rightarrow \boxed{U(h) = 15\left(\frac{23}{150}\right)^{h/10}}$$

(e) The profit of a company grows exponentially. The rate of growth is 5% per quarter. If this year's profit started at \$5000, write the profit of the company, $P(q)$ where q is quarter. Write the profit of the company $P(y)$ where y is years.

quarterly rate = 5% $\Rightarrow$ quarterly growth factor = 1.05

$$\Rightarrow \boxed{P(q) = 5000(1.05)^q}$$

4 quarters / year

$$\Rightarrow \boxed{P(y) = 5000(1.05)^{4y}}$$

(f) If a substance decays by 10% each hour, and the initial substance is 600 units. Write the function $F(t)$ where t is in days.

$r = 0.10 \Rightarrow a_h = 0.9$ (hourly decay factor)

$\Rightarrow a_d = (0.9)^{24}$ (daily decay factor)

$\Rightarrow \boxed{F(t) = 600(0.9)^{24t}}$

(g) You come up with the function

$$P(t) = 100(1.05)^t$$

where $P(t)$ is the population t years since 2000. Describe what 100 and 1.05 mean in the context of the problem. And state the growth rate, and explain what this means in the context of the problem.

100 represents the population in the year 2000.
The growth factor is 1.05, meaning that each year, the population grows by a factor of 1.05. The growth rate is 5%, so each year the population grows by 5%.

(h) You come up with the function

$$P(t) = 100(0.05)^t$$

where $P(t)$ is the population t years since 2000. Describe what 100 and 0.05 mean in the context of the problem. And state the decay rate, and explain what this means in the context of the problem.

100 is the population in 2000.
0.05 is the decay factor. Each year, the population decreases by a factor of 0.05.
The decay rate is 95%. Each year, the population is reduced by 95%.

37. Answer the following questions, by writing a function to describe the situation.

(a) A drug is eliminated from the body at a rate of 25% hourly. Write a function in terms of h, hours, if there was an initial amount of 40 mg.

$$D(h) = 40(1-0.25)^h$$

or $\boxed{D(h) = 40(0.75)^h}$

(b) A drug is eliminated from the body at a rate of 25% continuously. Write a function in terms of h, hours, if there was an initial amount of 40 mg.

$$\boxed{A(h) = 40e^{-0.25h}}$$

(c) A drug is eliminated from the body at a rate of 25% hourly. Write a function in terms of h, hours, if there was an initial amount of 400 mg.

$$\boxed{D(h) = 400(0.75)^h}$$

(d) A drug has a half-life of 5 hours. Write a function in terms of h, hours, if there is an initial value of 60 mg.

$$\boxed{D(h) = 60\left(\tfrac{1}{2}\right)^{h/5}}$$

(e) Write the above function as a continuous decay function.

hourly decay rate $= \left(\frac{1}{2}\right)^{1/5} \Rightarrow$ we need $e^r = \left(\frac{1}{2}\right)^{1/5}$

$\Rightarrow r = \frac{1}{5}\ln\left(\frac{1}{2}\right) \approx -0.1386$

$\Rightarrow \boxed{D(h) \approx 60e^{-0.1386h}}$

(f) If you have the exponential growth function of $500e^{0.456t}$, write this in the form Ca^t.

We have $C = 500$, $a = e^{0.456} \approx 1.5778$

$\Rightarrow \boxed{y = 500(1.5778)^t}$

(g) A population is increasing at a rate of 12% every 5 years. Write a function in terms of t, years, if there is an initial population of 400.

$$\boxed{y = 400(1.12)^{t/5}}$$

(h) A population is increasing at a continuous rate of 12%. Write a function in terms of t, years, if there is an initial population of 400.

$$\boxed{y = 400e^{0.12t}}$$

(i) A population has a doubling time of 50 years. Write a function in terms of t, years, if there is an initial population of 100.

$$\boxed{y = 100(2^{t/50})}$$

(j) Write the above function as a continuous growth function.

$$e^r = 2^{1/50} \Rightarrow r = \frac{1}{50}\ln(2) \approx 0.0139$$

$$\Rightarrow \boxed{y = 100e^{0.0139t}}$$

(k) If you have an exponential decay function of $500e^{-0.456t}$, write this in the form Ca^t.

$$a = e^{-0.456} \approx 0.6338$$

$$\Rightarrow \boxed{y = 500(0.6338)^t}$$

---

38. The two formulas are the only two that will be given on the exam:

$$A = P\left(1 + \frac{r}{n}\right)^{nt} \qquad A = Pe^{rt}$$

For example,

(a) If you invest \$100 in an account earning 5% compounded semi-annually. How much will you have at the end of 4 years?

$$A = 100\left(1 + \frac{0.05}{2}\right)^{2\cdot 4} = \boxed{\$121.84}$$

(b) If you invest \$100 in an account earning 5% compounded continuously. How much will you have at the end of 4 years?

$$A = 100\, e^{0.05 \cdot 4} = \boxed{\$122.14}$$

(c) If you want to have \$1000 at the end of 4 years, and you put this into an account earning 5% compounded semi-annually. How much should you invest at the start of 4 years?

$$1000 = P\left(1 + \frac{0.05}{2}\right)^{2 \cdot 4} \Rightarrow P = \frac{1000}{\left(1 + \frac{0.05}{2}\right)^8} = \boxed{\$820.75}$$

(d) If you want to have \$1000 at the end of 4 years, and you put this into an account earning 5% compounded continuously. How much should you invest at the start of 4 years?

$$1000 = Pe^{0.05 \cdot 4} \Rightarrow P = \frac{1000}{e^{0.05 \cdot 4}} = \boxed{\$818.73}$$

(e) If you want to have \$1000 at the end of 10 years, and you start with \$800. How much should your interest rate be, if it is compounded quarterly?

$$1000 = 800\left(1 + \frac{r}{4}\right)^{10 \cdot 4} \Rightarrow 1.25 = \left(1 + \frac{r}{4}\right)^{40}$$

$$\Rightarrow 1 + \frac{r}{4} = (1.25)^{1/40} \Rightarrow r = 4\,(1.25)^{1/40} - 4 \cong 0.0224$$

$$\Rightarrow \boxed{r = 2.24\,\%}$$

(f) How much should you start off with in an account that earns interest at 2.2% compounded daily, and you want \$200 more dollars in 10 years? (can be tricky, write everything that you know, and figure out what you need to solve for. Say, you start off with P dollars and end up with $P + 200$ dollars.)

$$P+200 = P\left(1+\frac{0.022}{365}\right)^{365\cdot 10}$$

$$\Rightarrow P\left[\left(1+\frac{0.022}{365}\right)^{3650} - 1\right] = 200$$

$$\Rightarrow P = \frac{200}{\left(1+\frac{0.022}{365}\right)^{3650} - 1} = \boxed{\$812.78}$$

(g) What is the effective interest rate if you invest in an account that earns 6.5% daily? continuously?

$$\text{Daily: } r_{eff} = \left(1+\frac{0.065}{365}\right)^{365} - 1 = 0.0672$$

$$\Rightarrow \boxed{r_{eff} = 6.72\%}$$

$$\text{Continuously: } r_{eff} = e^{0.065} - 1 = \boxed{6.72\%}$$

(h) What is the annual and semi annual nominal interest rate if the effective interest rate is 5.7%?

$$\text{annual: } \boxed{r = 5.7\%}$$

$$\text{Semi annual: } 0.057 = \left(1+\frac{r}{2}\right)^2 - 1$$

$$\Rightarrow \left(1+\frac{r}{2}\right)^2 = 1.057$$

$$\Rightarrow 1+\frac{r}{2} = \sqrt{1.057}$$

$$\Rightarrow r = 2\sqrt{1.057} - 2 = \boxed{5.62\%}$$

39. Using the rule of 70 answer the following questions:

(a) If a population has a growth rate of 5% per year, will this have a doubling time or half-life, explain? Approximate this.

Doubling time, due to growth.

$$\approx \frac{70}{5} = \boxed{14 \text{ years}}$$

(b) If a population has a decay rate of 5% per year, will this have a doubling time or half-life, explain? Approximate this.

Half-Life, due to decay

$$\approx \frac{70}{5} = \boxed{14 \text{ years}}$$

(c) According to this function

$$P(t) = 500(1.20)^t$$

where t is in years, will this function have a doubling time or half-life, explain? Approximate this.

Doubling time, since it is a growth model.

$R = 20\%$

$$\Rightarrow \text{Doubling Time} \approx \frac{70}{20} = \boxed{3.5 \text{ years.}}$$

(d) According to this function

$$P(t) = 500(0.20)^t$$

where t is in days, will this function have a doubling time or half-life, explain? Approximate this.

Half-Life, since P is a decay model

$R = 80\%$

$$\Rightarrow \text{Half-Life} \approx \frac{70}{80} = \boxed{0.875 \text{ days}}$$

40. The following table gives the number of U.S. jobs supported by exports to Mexico for recent years and can be found in Glassman. Number is in thousands. For your regression function let $t = 0$ correspond to year 1986. (With this information you will need to adjust your table prior to inputting in calculator.)

Year	Jobs
1986	274
1987	300
1988	400
1989	500
1990	590
1991	700
1992	900

(a) Write the best fit function (either linear or exponential), round to 4 decimals.

Exponential regression is the best fit (check the r-values)

$\boxed{y = 263.4969(1.2236)^x}$

(b) What is the correlation coefficient (round to 4 decimals)? And what two things does this tell us?

$\boxed{r = 0.9959}$ It tells us that the function is a good fit to the data, and that this is a growth model (positive correlation).

(c) State the two numerical values in part (a) and explain what they represent in the context of the problem.

263.4969 is the model's prediction of the number of U.S. jobs supported by exports to Mexico, in thousands, in the year 1986.

1.2236 is the growth factor. This means that the model predicts that the number of U.S. jobs supported by exports to Mexico grows at a rate of 22.36% per year.

41. Consider the following table which describes a cat population t years after 2000:

Year (t)	Cat Population
0	100
2	200
4	380
6	753
8	1477
10	2895

(a) What is the best fit exponential function? (round to 4 decimals).

$\boxed{y = 100.4921(1.3991)^x}$

(b) Why is the initial value in the regression function 100.4921 not the same as the table's initial value, 100? What does this say about our regression function?

Because the regression function tries to minimize the distance from the curve to each of the data points, so it ends up sacrificing some accuracy with known values. It says that the regression function is not a perfect fit.

(c) What is the correlation coefficient (round to 5 decimals)? Does this say our *data* is accurate? If not, what does this indicate is accurate?

$r = 0.99994$. It does not say that the data is accurate, it says that the regression function is an accurate representation of the data.

(d) In our above function, is t every year, or every 2?

every year

(e) What is the growth rate? What does this tell us?

The growth rate is 39.91%. This says that the model predicts a growth of 39.91% per year in the population of cats.

42. True or False. (If False know what was done wrong.) Note, the same true properties hold for ln. I'm not rewriting these problems, but we could rewrite as ln.

(a) $\log_{10} x = 10^x$ F

(b) $\log_{10}(x+1) = \log(x+1)$ T

(c) $\log(x+y) = \log x + \log y$ F

(d) $\log(x+y) = \log x + y$ F

(e) $\log(x+y) = \log x \cdot \log y$ F

(f) $3\log x = \log x^3$ T

(g) $2\log(x+y) = \log(x^2+y^2)$ F

(h) $\log x^2 = \log x + \log x$ T

(i) $\log x^2 = (\log x)^2$ F

(j) $\log\left(\frac{x}{y}\right) = \frac{\log x}{\log y}$ F

(k) $\log x - \log y = \frac{\log x}{\log y}$ F

(l) $\log x - \log y = \log\left(\frac{x}{y}\right)$ T

(m) $\log x - \log y = \frac{\log x}{y}$ F

(n) $\log(xy) = \log x \cdot \log y$ F

(o) $\log(x^2+4x+4) = 2\log(x+2)$ T

43. Solve the following equations for the variable. Make sure you can solve *exactly* and solve rounding to the proper decimal. (Note, some may not exist or be messy. Note, you cannot take the natural log, or log of a negative number.)

(a) $6^x - 10^{3x+2} = 0$

$6^x = 10^{3x+2} \cancel{\Rightarrow x\ln(6) = (3x+2)(\ln(10))}$

use $\log_{10}$

$x\log 6 = 3x+2 \Rightarrow x(3-\log 6) = -2$

$\Rightarrow \boxed{x = -\frac{2}{3-\log 6}}$

(b) $10^{x^2} = 100^{x^2-6}$

$100 = 10^2$

$\Rightarrow 10^{x^2} = (10^2)^{x^2-6} \Rightarrow 10^{x^2} = 10^{2x^2-12}$

$\Rightarrow x^2 = 2x^2 - 12$

$\Rightarrow x^2 = 12$

$\Rightarrow \boxed{x = \pm 2\sqrt{3}}$

(c) $e^{x+8} = 10^x$

$$x+8 = x\ln(10) \Rightarrow x(\ln 10 - 1) = 8$$

$$\Rightarrow \boxed{x = \frac{8}{\ln 10 - 1}}$$

(d) $1005 = 123e^x$ $\Rightarrow e^x = \frac{1005}{123}$

$$\Rightarrow \boxed{x = \ln\left(\frac{1005}{123}\right)}$$

(e) $1005 = 123(x)^4$ $\Rightarrow x^4 = \frac{1005}{123}$

$$\Rightarrow \boxed{x = \pm \sqrt[4]{\frac{1005}{123}}}$$

(f) $8^x - 2^{x+6} = 0$ $\Rightarrow (2^3)^x - 2^{x+6} = 0$

$$\Rightarrow 2^{3x} = 2^{x+6} \Rightarrow 3x = x+6$$

$$\Rightarrow 2x = 6$$

$$\Rightarrow \boxed{x = 3}$$

(g) $5e^{2x+5} = 100$

$$e^{2x+5} = 20$$

$$\Rightarrow 2x+5 = \ln(20)$$

$$\Rightarrow \boxed{x = \frac{\ln(20) - 5}{2}}$$

44. If possible expand using the properties of logarithms, if not possible state why.

(a) $\ln(x^3\sqrt{yz})$ $= \ln(x^3) + \ln(\sqrt{yz})$

$= 3\ln x + \frac{1}{2}\ln(yz)$

$= 3\ln x + \frac{1}{2}(\ln y + \ln z)$

$= \boxed{3\ln x + \frac{1}{2}\ln y + \frac{1}{2}\ln z}$

(b) $\ln\left(\frac{x^3}{\sqrt{yz}}\right)$

$= \ln(x^3) - \ln(\sqrt{yz})$

$= \ln(x^3) - \frac{1}{2}\ln(yz) = 3\ln x - \frac{1}{2}(\ln y + \ln z)$

$= \boxed{3\ln x - \frac{1}{2}\ln y - \frac{1}{2}\ln z}$

(c) $\ln\left(\frac{x^3}{\sqrt{y}}\sqrt{z}\right)$ $= \ln\left(\frac{x^3}{\sqrt{y}}\right) + \ln(\sqrt{z})$

$= \ln(x^3) - \ln(\sqrt{y}) + \ln(\sqrt{z})$

$= \boxed{3\ln x - \frac{1}{2}\ln y + \frac{1}{2}\ln z}$

(d) $5\ln(x+y)$ Cannot expand. No properties for sums inside of a log.

(e) $5\ln\left(\sqrt{x}\cdot\frac{z}{\sqrt[3]{y}}\right)$ $= 5\ln(\sqrt{x}) + 5\ln\left(\frac{z}{\sqrt[3]{y}}\right)$

$= \frac{5}{2}\ln x + 5(\ln z - \ln(\sqrt[3]{y}))$

$= \boxed{\frac{5}{2}\ln x + 5\ln z - \frac{5}{3}\ln(y)}$

45. Contract, rewriting the expression as a single logarithm.

(a) $\frac{1}{2}\ln x - \frac{3}{2}\ln(x+1)$ $= \ln(\sqrt{x}) - \ln\left(\sqrt{(x+1)^3}\right)$

$$= \boxed{\ln\left(\sqrt{\frac{x}{(x+1)^3}}\right)}$$

(b) $3\ln(x) + 5\ln(y) - \ln(z)$ $= \ln(x^3) + \ln(y^5) - \ln(z)$

$$= \boxed{\ln\left(\frac{x^3y^5}{z}\right)}$$

(c) $4(\ln x + \ln y - \ln z)$ $= 4\ln\left(\frac{xy}{z}\right)$

$$= \boxed{\ln\left[\left(\frac{xy}{z}\right)^4\right]}$$

(d) $\ln(z+5) - \ln(z-5)$ $= \boxed{\ln\left(\frac{z+5}{z-5}\right)}$

---

46. Answer the following financial problems.

(a) If you want to have \$1000 at the end of 10 years, and you start with \$800. How much should your interest rate be, if it is compounded continuously?

$$1000 = 800e^{r\cdot 10} \Rightarrow 1.25 = e^{10r}$$

$$\Rightarrow 10r = \ln(1.25)$$

$$\Rightarrow r = \frac{\ln(1.25)}{10} \approx 0.0223$$

$$\Rightarrow \boxed{r = 2.23\%}$$

(b) How many years will it take \$500 to grow to \$600 with an interest rate of 3% compounded monthly?

$$600 = 500\left(1+\frac{0.03}{12}\right)^{12t} \Rightarrow 1.2 = \left(1+\frac{0.03}{12}\right)^{12t}$$

$$\Rightarrow 12t \ln\left(1+\frac{0.03}{12}\right) = \ln(1.2)$$

$$\Rightarrow t = \frac{\ln(1.2)}{12\ln\left(1+\frac{0.03}{12}\right)} \cong \boxed{6.0850 \text{ years}}$$

(c) How many years will it take \$500 to grow to \$600 with an interest rate of 3% compounded continuously? (Round to the whole year)

$$600 = 500e^{0.03t} \Rightarrow 1.2 = e^{0.03t}$$

$$\Rightarrow 0.03t = \ln(1.2)$$

$$\Rightarrow t = \frac{\ln(1.2)}{0.03} \cong \boxed{6.0774 \text{ years}}$$

---

47. Let $f(t)$ be the quantity of ampicillin, in mg, in the bloodstream at time t hours since the drug was taken. At $t = 0$, the amount is 250 mg, and approximately 42% of the drug is eliminated each hour.

(a) Write the exponential function, $f(t)$ for the amount of drug in the system.

$$f(t) = 250(0.58)^t$$

(b) Find exactly how many hours (rounding to two decimals) it will take for there to be 100 mg of the drug in the bloodstream. Showing all algebraic work.

$$100 = 250(0.58)^t$$

$$\Rightarrow 0.4 = 0.58^t \Rightarrow t\ln(0.58) = \ln(0.4)$$

$$\Rightarrow t = \frac{\ln(0.4)}{\ln(0.58)} \approx \boxed{1.68 \text{ hours}}$$

(c) What if the drug was continuously eliminated at a rate of 42%? What is the exponential function then?

$$\boxed{f(t) = 250e^{-0.42t}}$$

(d) How many hours (rounding to two decimals) will it take for there to be 100 mg of the drug in the bloodstream for your function in part (c). Show all algebraic work.

$$100 = 250e^{-0.42t} \Rightarrow 0.4 = e^{-0.42t}$$

$$\Rightarrow -0.42t = \ln(0.4)$$

$$\Rightarrow t = \frac{\ln(0.4)}{-0.42} \approx \boxed{2.18 \text{ hours}}$$

---

48. Answer the following questions (not using the rule of 70 but solving exactly).

(a) If a population has a growth rate of 5% per year, will this have a doubling time or half-life? Explain and find this. Doubling time, due to growth

$$2 = 1.05^t \Rightarrow t\ln(1.05) = \ln(2)$$

$$\Rightarrow t = \frac{\ln(2)}{\ln(1.05)} \approx \boxed{14.2067 \text{ years}}$$

(b) If a population has a decay rate of 5% per month, will this have a doubling time or half-life? Explain and find this. Half-life, due to decay.

$$\frac{1}{2} = 0.95^t \Rightarrow t\ln(0.95) = \ln(1/2)$$

$$\Rightarrow t = \frac{\ln(1/2)}{\ln(0.95)} \approx \boxed{13.5134 \text{ months}}$$

(c) According to this function $P(t) = 500(1.20)^t$ where t is in days. Will this function have a doubling time or half-life? Explain and find this.

Doubling time, because this is a growth model

$$2 = 1.20^t$$

$$\Rightarrow \boxed{t = \frac{\ln(2)}{\ln(1.20)} \approx 3.8018 \text{ days}}$$

(d) According to this function $P(t) = 500(0.20)^t$ where t is in hours. Will this function have a doubling time or half-life. Explain and find this.

Half-Life, because this is a decay model.

$$\frac{1}{2} = 0.20^t \Rightarrow t = \frac{\ln(1/2)}{\ln(0.20)} \approx \boxed{0.4307 \text{ hours}}$$

(e) Find the exact doubling time for a substance that earns increases 5% per quarter year. (Round to the nearest quarter year.)

$$2 = 1.05^t \Rightarrow t = \frac{\ln(2)}{\ln(1.05)} = \boxed{14 \text{ quarter years}}$$

(f) Find the exact half-life for a radioactive substance that has an annual decay rate of 2.4%. (Round to the nearest year.)

$$\frac{1}{2} = 0.976^t \Rightarrow t = \frac{\ln(1/2)}{\ln(0.976)} \approx 28.53$$

or $\boxed{t = 29 \text{ years}}$

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49. Answer the following questions about logarithms.

(a) What is the domain of $\log x$? $(0, \infty)$

(b) What is the range of $\log x$? $(-\infty, \infty)$.

(c) Is there a horizontal asymptote? Where is this located? No

(d) Is there a vertical asymptote? Where is this located? Yes, at $x = 0$.

(e) What is the difference between $\log x$ and $\ln x$? $\log x$ is the ;

50. Solve the following exactly. (Keep in mind the domain of log and ln.)

(f) $\ln(x) - \ln(x-1) = 2$

$$\ln\left(\frac{x}{x-1}\right) = 2 \Rightarrow \frac{x}{x-1} = e^2$$

$$\Rightarrow x = e^2x - e^2$$

$$\Rightarrow x(e^2-1) = e^2 \Rightarrow \boxed{x = \frac{e^2}{e^2-1}}$$

(g) $\log(x) - \log(x-1) = 2$

$$\log\left(\frac{x}{x-1}\right) = 2$$

$$\Rightarrow \frac{x}{x-1} = 10^2 = 100$$

$$\Rightarrow x = 100x - 100 \Rightarrow 99x = 100$$

$$\Rightarrow \boxed{x = \frac{100}{99}}$$

(h) $\log(x) - \log(x+3) = 1$

$$\log\left(\frac{x}{x+3}\right) = 1$$

$$\Rightarrow \frac{x}{x+3} = 10 \Rightarrow x = 10x + 30 \Rightarrow 9x = -30$$

$$\Rightarrow \cancel{x = -\frac{30}{9}}$$ $\boxed{\text{No solution}}$

(can't have a negative in the log)

(i) $\ln(x^2+3) - \ln(x^2) = 2$

$$\ln\left(\frac{x^2+3}{x^2}\right) = 2$$

$$\Rightarrow \frac{x^2+3}{x^2} = e^2$$

$$\Rightarrow x^2+3 = e^2x^2$$

$$\Rightarrow x^2(e^2-1) = 3$$

$$\Rightarrow x^2 = \frac{3}{e^2-1}$$

$$\Rightarrow \boxed{x = \sqrt{\frac{3}{e^2-1}}}$$

(only the positive value works here)

51. Suppose I am making a pasta sauce with tomatoes and wine (and a few other things that aren't going to impact our pH much). Wine has a pH of 4 and tomatoes a pH of 5.

(a) The directions call for 1.75 cups of tomatoes and 0.25 cups of wine (who knows how good this sauce will be?!). What percent of the mixture (if I only consider tomatoes and wine) is using the tomatoes and what percent is using the wine? (Hint: Remember to find the percent consider the total amount of the item you are considering divided by the total amount you have.)

tomatoes: $\frac{1.75}{2.00} = \boxed{87.5\%}$

wine: $\frac{0.25}{2.00} = \boxed{12.5\%}$

(b) What is the percent of tomatoes in decimal form? And the percent of wine in decimal form?

tomatoes: 0.875 wine: 0.125

(c) What is the *exact* hydrogen ion concentration of my sauce?

wine: $4 = -\log[H^+] \Rightarrow [H^+] = 10^{-4}$

tomatoes: $5 = -\log[H^+] \Rightarrow [H^+] = 10^{-5}$

Our sauce

$$0.875 \cdot 10^{-5} + 0.125 \cdot 10^{-4} = 2.125 \times 10^{-5}$$

$$\Rightarrow \boxed{[H^+] = 2.125 \times 10^{-5}}$$

(d) What is the pH of my sauce? (Rounding to two decimals.)

$$pH = -\log[H^+] = -\log(2.125\times10^{-5}) = \boxed{4.67}$$

(e) If I want my pH of my sauce to be 4.75 exactly. What percent of tomatoes and what percent of wine should I include? (Rounding to two decimals.)

p = decimal form of percent of tomatoes

$$\Rightarrow [H^+] = p\cdot10^{-5} + (1-p)\cdot10^{-4} \quad \text{If } pH = 4.75 \Rightarrow [H^+] = 10^{-4.75}$$

$$\Rightarrow 10^{-4.75} = p\cdot10^{-5} + (1-p)\cdot10^{-4} = p\cdot10^{-5} + 10^{-4} - p\cdot10^{-4}$$

$$\Rightarrow p(10^{-5} - 10^{-4}) = 10^{-4.75} - 10^{-4} \Rightarrow p = \frac{10^{-4.75} - 10^{-4}}{10^{-5} - 10^{-4}} = 0.9135$$

$\Rightarrow$ $\boxed{\text{91.35\% tomatoes, 8.65\% wine}}$

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52. If sound doubles in intensity, by how many units does its decibel rating increase? Show all mathematical work.

$$N = 10\cdot\log(I/I_0)$$

I doubles $\Rightarrow I_2 = 2I_1$

$$N_1 = 10\cdot\log\left(\frac{I_1}{I_0}\right), \quad N_2 = 10\cdot\log\left(\frac{I_2}{I_0}\right) = 10\cdot\log\left(\frac{2I_1}{I_0}\right)$$

$$\Rightarrow N_2 - N_1 = 10\cdot\log\left(\frac{2I_1}{I_0}\right) - 10\cdot\log\left(\frac{I_1}{I_0}\right)$$

$$= 10\left(\log\left(\frac{2I_1}{I_0}\right) - \log\left(\frac{I_1}{I_0}\right)\right)$$

$$= 10\log\left(\frac{2\cancel{I_1}}{\cancel{I_0}}\cdot\frac{\cancel{I_0}}{\cancel{I_1}}\right) = 10\cdot\log(2)$$

$\Rightarrow$ $\boxed{\text{The decibel rating increases by } 10\cdot\log(2) \text{ units}}$ ≈ 3 units