# EXAM 2 STUDY GUIDE

MATH 223 · FALL 2012

## 1. Concepts

**1.** (a) Write expressions for the partial derivatives $f_x(a,b)$ and $f_y(a,b)$ as limits.
(b) How do you interpret $f_x(a,b)$ and $f_y(a,b)$ geometrically? How do you interpret them as rates of change?
(c) If $f(x,y)$ is given by a formula, how do you calculate $f_x$ and $f_y$?

**2.** Under what conditions can you conclude that $f_{xy} = f_{yx}$?

**3.** How do you find a tangent plane to each of the following types of surfaces?
(a) A graph of a function of two variables, $z = f(x,y)$
(b) A level surface of a function of three variables, $F(x,y,z) = c$

**4.** Define the local linearization of $f$ at $(a,b)$. What is the corresponding linear approximation? What is the geometric interpretation of the linear approximation?

**5.** If $z = f(x,y)$, what are the differentials $dx$, $dy$ and $dz$?

**6.** State the chain rule for the case where $z = f(x,y)$ and $x$ and $y$ are functions of one variable. What if $x$ and $y$ are functions of two variables? Now, state the chain rule for the case that $z = g(x,y,t)$, and $x$ and $y$ are both functions of $t$.

**7.** If $z$ is defined implicitly as a function of $x$ and $y$ by an equation of the form $F(x,y,z) = 0$, how do you find $\partial z/\partial x$ and $\partial z/\partial y$?

**8.** (a) Write an expression as a limit for the directional derivative of $f$ at $(x_0,y_0)$ in the direction of a unit vector $\hat{u} = a\hat{\imath} + b\hat{\jmath}$. How do you interpret it as a rate? How do you interpret it geometrically?
(b) If $f$ has continuous first order partial derivatives, write an expression for $f_{\hat{u}}(x_0,y_0)$ in terms of $f_x$ and $f_y$.

**9.** (a) Define the gradient vector grad$f$ for a function $f$ of two or three variables.
(b) Express $f_{\hat{u}}$ in terms of grad$f$.

(c) Explain the geometric significance of the gradient.

**10.** What do the following statements mean?
(a) $f$ has a local maximum at $(a, b)$.
(b) $f$ has a global maximum at $(a, b)$.
(c) $f$ has a local minimum at $(a, b)$.
(d) $f$ has a global minimum at $(a, b)$.
(e) $f$ has a saddle point at $(a, b)$.

**11.** (a) If $f$ has a local maximum at $(a, b)$, what can you say about its partial derivatives at $(a, b)$?
(b) What is a critical point of $f$?

**12.** State the second derivative test.

**13.** (a) Let $\mathbb{R}^2$ denote the set of all points $(x, y)$, where $x$ and $y$ are real numbers. What is a closed set in $\mathbb{R}^2$? What is a bounded set?
(b) State the Extreme Value Theorem for functions of two variables.
(c) How do you find the values that the Extreme Value Theorem guarantees?

**14.** Suppose that $f$ is a continuous function defined on a rectangle $R : a \leq x \leq b, c \leq y \leq d$.
(a) Write an expression for a double Riemann sum of $f$. If $f(x, y) \geq 0$, what does the sum represent?
(b) Write the definition of $\int_R f dA$ as a limit.
(c) What is the geometric interpretation of $\int_R f dA$ if $f(x, y) \geq 0$? What if $f$ takes on both positive and negative values?
(d) How do you evaluate $\int_R f dA$?
(e) Write an expression for the average value of $f$.

**15.** How do you define $\int_D f dA$ if $D$ is a bounded region that is not a rectangle?

**16.** If $D$ is a bounded region, write an expression representing the area $D$ in terms of a double integral.