EXAM 2 STUDY GUIDE

MATH 223 · FALL 2012

2. Exercises

Partial Derivatives

You should be able to confidently embark on exercises which test your fundamental understanding to the concept of partial derivative, such as the following:

Exercise 1. The temperature T at a location in the Northern Hemisphere depends on the longitude x, the latitiude y, and time t, so we can write $T = f(x, y, t)$. Let's measure time in hours from the beginning of January.

(a) What are the meanings of the partial derivatives $\partial T/\partial x$, $\partial T/\partial y$, and $\partial T/\partial t$?

(b) Honolulu has longitude 158°W and latitude 21°N. Suppose that at 9 : 00 A.M. on January 1 the wind is blowing hot air to the northeast, so the air to the west and south is warm and the air to the north and east is cooler. Would you expect $f_x(158, 21, 9)$, $f_y(158, 21, 9)$ and $f_t(158, 21, 9)$ to be positive or negative? Explain.

You should also be able to calculate partial derivatives efficiently;

Exercise 2. Find the first partial derivatives of the function.

(a)

$$z = y \ln x$$

(b)

$$f(s, t) = \frac{st^2}{(s^2 + t^2)},$$

(c)

$$f(x, t) = \arctan\left(x\sqrt{t}\right),$$

(d)

$$f(x, y) = \int_x^y \cos\left(t^2\right) dt,$$

(e)

$$u = x^{y/z},$$

(f)

$$f(u, v, w) = w \tan(uv)$$

Do the following exercises on pages 728-729: # 9-12, 16, 17, 19, 20, 22.

Tangent planes and linear approximation.

Exercise 3. Find an equation of the tangent plane to the given surface at the specified point.

$z = e^{x^2 - y^2}$ at the point $(1, -1, 1)$.

For the following exercise, do the graphing part if you have access to sufficient graphing instruments for functions of two variables.

Exercise 4. Graph the surface and the tangent plane at the given point. (Choose the domain and viewpoint so that you get a good view of both the surface and the tangent plane.) Then zoom in until the surface and the tangent plane become indestinguishable.

$z = \arctan(xy^2)$, at the point $(1, 1, \pi/4)$.

Exercise 5. Find the local linearization $L(x, y)$, of the function at the given point.

(a) $f(x, y) = x/y$, at $(6, 3)$.

(b) $f(x, y) = \sqrt{x + e^{4y}}$ at the point $(3, 0)$.

Exercise 6. Find the lienar approximation of the function

$$f(x, y) = \ln(x - 3y)$$

at the point $(7, 2)$ and use it to approximate $f(6.9, 2.06)$.

Exercise 7. Find the differentials of the following functions.

(a) $v = y \cos xy$

(b) $u = r/(s + 2t)$.

(c) $w = xye^{xz}$.

Exercise 8. A boundary stripe 3 in. wide is painted around a rectangle whose dimensions are 100 ft. by 200 ft. Use differentials to approximate the number of square feet of paint in the stripe.

Do the following exercises from pages 741-742 in the textbook: # 21, 26, 27.

Gradients and directional derivatives.

Exercise 9. Suppose that over a certain region of space the electrical potential V is given by $V(x, y, z) = 5x^2 - 3xy + xyz$.

(a) Find the rate of change of the potential at $P(3, 4, 5)$ in the direction of the vector $\overrightarrow{v} = \hat{i} + \hat{j} - \hat{k}$.

(b) In which direction does V change most rapidly at P (give your answer as a direction that makes an angle θ with the horizontal).

(c) Find the maximum rate of change of V at P.

Exercise 10. Suppose you are climbing a hill whose shape is given by the equation $z = 1000 - 0.01x^2 - 0.02y^2$, where x, y, z are measured in meters, and you are standing at a point with coordinates $(50, 80, 847)$. The positive x-axis points east and the positive y-axis points north.

(a) If you walk due south, will you start to ascend or descend? At what rate?

(b) If you walk northwest, will you start to ascend or descend? At what rate?

(c) In which direction is the slope largest? What is the rate of axcent in that direction? At what angle above the horizontal does the path in that direction begin?

Exercise 11. If $g(x,y) = x - y^2$, find the gradient vector $\nabla g(3,-1)$ and use it to find the tangent line to the level curve $g(x,y) = 2$ at the point $(3,-1)$. Sketch the level curve, the tangent line, and the gradient vector.

Do the following exercises from the textbook on pages 748-750: # 1-14, 16, 18, 20, 30-35, 48, 53-57, 63, 71, 72.

Do the following exercises from the textbook on pages 756-758: # 1-12, 13 - 18, 19, 21, 23, 31, 32, 33, 38, 39, 40, 45, 50, 54, 58.

The Chain Rule: To begin, do the following exercises from the textbook, pages 765-767: # 1-6, 7-14, 16, 21, 22, 23, 24, 38-40.

Exercise 12. The temperature at a point (x,y) is $T(x,y)$, measured in degrees Celsius. A bug crawls so that its position after t seconds is given by $x = \sqrt{1+t}$, $y = 2+\frac{1}{3}t$, where x and y are measured in centimeters. The termperature function satisfies $T_x(2,3) = 4$, $T_y(2,3) = 3$.

How fast is the temperature rising on the bug's path after 3 seconds?

Exercise 13. The length, l, width w, and height h of a box change with time. At a certain instant the dimensions are $l = 1$ m and $w = h = 2$ m, and l and w are increasing at a rate of 2 m/s while h is decreasing at a rate of 3 m/s. At that instant, find the rates at which the following quantities are changing.

(a) The volume. (b) The surface area. (c) The length of a diagonal.

Maximum and minimum values:

I am going to combine sections 14.7, 15.1 and 15.2 into the basic idea of finding maximum and minimum values, both local and global, for given functions. Section 14.7 really just covers second order partial derivatives, which we already know how to calculate, so I won't give you any review for that. Section 15.1 and 15.2 are the main key points here. Good problems in 15.2 cover the necessary tools covered in 15.1, so that section will be mostly stressed here.

From section 15.1, do the following exercises from the book: # 1, 2, 7-16, 21, 22, 23, 28.

Exercise 14. Find the absolute maximum and minimum values of f on the region R

(a) $f(x,y) = 4x + 6y - x^2 - y^2$, $R = \{(x,y) : 0 \leq x \leq 4, 0 \leq y \leq 5\}$.

(b) $f(x,y) = x^4 + y^4 - 4xy + 2$, $R = \{(x,y) : 0 \leq x \leq 3, 0 \leq y \leq 2\}$.

Exercise 15. Find the shortest distance from the point $(2,1,-1)$ to the plane $x+y-z = 1$.

Do the following exercises from the book, pages 806-808 : # 4, 5-7, 8, 9-13, 15, 17, 18, 19, 21, 22

The definite integral for functions of two variables.

For the review of this section, simply do the following exercises in the book, pages 831 - 833 # 1, 5, 6, 7, 8, 9-24, 25, 26, 38, 29, 30.