Review:

These document provides you a list of questions that you should be able to complete by the end of the semester. You should also be able to define any terms on the 'Key Ideas' sheet also provided for you. (Some terms may not be tested on this sheet.)

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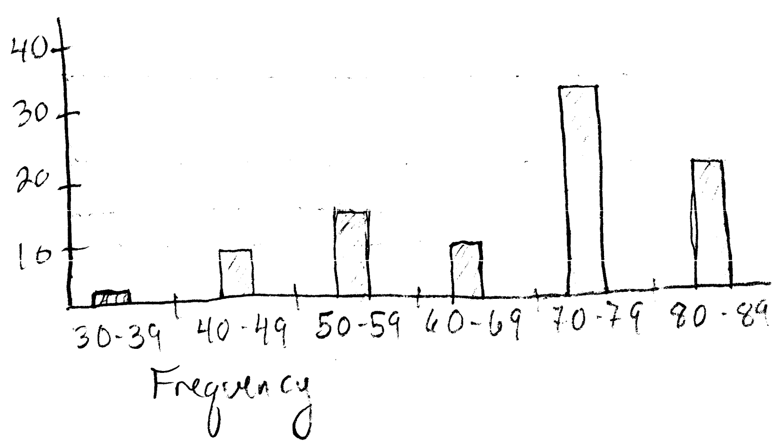
1. Using the data on page 15 of your Supplements book (Countries from Algeria to Haiti... *not* onto page 16) efficiently turn this data into a frequency and relative frequency table, given below:

Life Expectancy	Frequency of Countries	Relative Frequency of Countries
30 – 39	1	1.0989%
40 – 49	9	9.8901%
50 – 59	15	16.4835%
60 – 69	10	10.9890%
70 – 79	34	37.3626%
80 – 89	22	24.1758%
Total :	91	

Draw two histograms to represent this data (one with frequency, one with relative frequency):



(The chart for relative frequency is similar)

Explain in your own words what is the 'best' way (what does 'best' mean?) to represent this data, table given in Supplements book, table you provided above or histogram? And why is this the 'best'? (There is no 'right' answer, but make sure your reasoning supports your answer–this could be wrong.)

Answers will vary. Decide for yourself

2. Below is a list of some winter Olympic sports, and the number of people surveyed who favor the sport.

Sport	Number of People
Alpine Skiing	100
Biathlon	200
Curling	153
Ice Hockey	255
Speed Skating	96

Total = 804

(a) What is the frequency and relative frequency of the number of people who favor curling? Speed skating?

Curling: frequency = 153
Rel. freq. = 19.03%

SS: freq. = 96
Rel. freq. = 11.94%

(b) Can you state which sport has the highest relative frequency, just given the list of 'number of people' and without calculations? Why or why not?

Yes, since frequency and relative frequency are proportional,

Answer = Ice Hockey

3. Answer the following questions concerning different types of data:

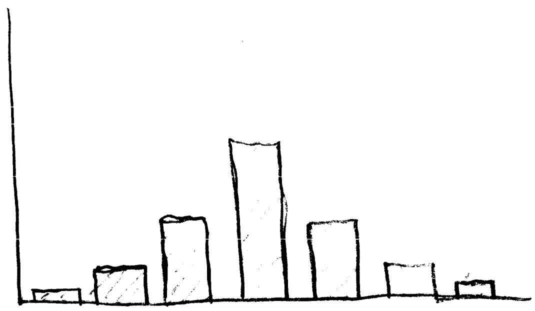
(a) If you have data that is skewed left, which will be greater, mean or median and why?

The median will be greater, because there are outliers to the left, bringing down the mean.

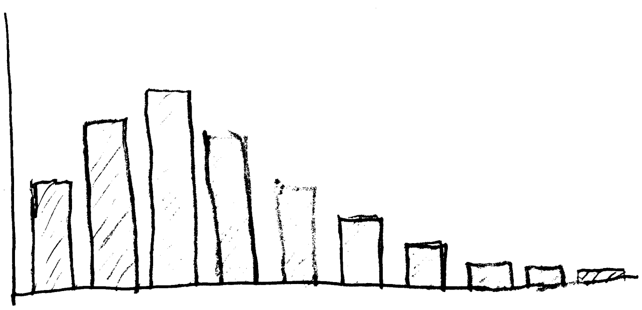
(b) If you have data that is skewed right, which will be greater, mean or median and why?

The mean will be greater, because there are outliers to the right, bringing up the mean.

(c) Draw an example of a bar chart of data that is unimodal.



(d) Draw an example of a bar chart of data that is skewed right.



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4. Using the data on page 9 in the Supplements book (page is titled 'The Mean Truth about Means') answer the following questions:

(a) Using the top 10 countries, rounding to the nearest million (i.e. China would be 1,339,000,000 not 1,338,612,968) find the mean. It is fine to not write all mathematical work, but explain the process you used in order to state the mean.

$$\text{mean} = \frac{\text{sum of populations of top 10 Countries}}{10} = \boxed{399{,}900{,}000}$$

(b) State the median. Again, explain the process you used in order to solve this problem.

$$\text{median} = \frac{\text{pop. of Brazil} + \text{pop. of Pakistan}}{2} = \boxed{187{,}500{,}000}$$

(c) Why would one use the mean?

To make the average population seem very big.

(d) Why would one use the median?

To give a better picture on what the majority of the most populous countries are like.

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5. I'm training for a race, and have ran a few days. My mileage, for the past 5 runs has been

$$2.19, 2.30, 4.10, 2.32, 2.25$$

If I wanted to look as impressive as I could, would I use the mean or median to describe my 'average' runs? Why?

mean = 2.632 , median = 2.30

You would use the mean to be more impressive, because it is higher, so you look like you run more on average.

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6. Answer the following questions: Given these four test results. The numbers given are the number of problems correct, out of 25

$$18, 15, 22, 20.$$

(a) What is the mean?

$$\text{mean} = \frac{18+15+22+20}{4} = \boxed{18.75}$$

(b) What is the median?

$$\text{median} = \frac{18+20}{2} = \boxed{19}$$

(c) Would you rather state your mean or median as your average and why?

median, because it makes me look better.

(d) If you wanted a mean of 80% on five exams what would you want the number correct to be (out of 25 questions) for your fifth exam? First, 80% of 25 is 20, so

I want my mean to be 20. Let x = score on my fifth exam.

Then $\dfrac{18+15+22+20+x}{5} = 20$

$\Rightarrow 18+15+22+20+x = 100 \Rightarrow \boxed{x = 25}$

(e) If you wanted a median of 80% on five exams what would you want the number correct to be (out of 25 questions) for your fifth exam? (List all possible, if there is more than one possibility)

If my 5th exam is a 20, 21, 22, 23, 24 or 25, then my median will be 20 (80%).

(f) With any number of tests, is it ever possible to have a mean of exactly 100%? Why or why not?

Only if you score 100% on every single test. Otherwise, the lower test scores will always drag it down.

(g) With any number of tests, is it ever possible to have a median of exactly 100%? Why or why not?

Same answer as in part (f).

7. Know how to tell if a point is a solution of a function (given the function, or graph of function). This can get tricky, if you are asked to find $f(5)$, what is 5? The x-value or y-value? Homework has lots of examples of this, if you do not understand, go through these examples, and/or come to my office to talk about it.

8. If you were to be asked to write h, height, as a function of t, time, what is the $x-$ and what is the $y-$ value? t is the x-value, h is the y-value.

9. If the height of an animal increases by 6 inches, every year, write height, h, as a function of years, t, when the animal was born with a height of 2 inches.

$$\boxed{h = \underbrace{2}_{\text{initial value}} + \underbrace{6}_{\text{rate of change}}t}$$

10. Given the below scenarios state if the description is a function and if so state the input and output of these, if not state why not a function:

(a) The total snowfall for each day on a given month.

yes, day of month = input,
total snowfall = output

(b) The highest exam score for each student in this class.

Yes, student = input
highest exam score = output

(c) The exam scores for each student in this class at the end of the year.

No, because each student has multiple
exam scores (presumably)

11. Given the following linear functions and descriptions. State the units and what the slope and y-intercept represent in the context of the problem.

(a) The relationship between the balance, B, in dollars, remain on a mortgage loan and the number, n, of monthly payments which you've made is given by the linear function

$$B = 403,200 - 1120n.$$

Slope: For each monthly payment, my loan balance decreases by $1,120.

Y-intercept: My initial loan balance is $ 403,200

(b) Annual Income in dollars, d, is a function of members of household, n, given by the function

$$d = 6438n + 12451$$

Slope: For each additional member of your household, your annual income increases by $6,438.

Y-intercept: A household with nobody in it has an annual income of $12,451.

(c) The average female math SAT scores, m, is a function of years since 2000, t, and can be represented as

$$m = \frac{1}{8}t + 498.$$

Slope: Since 2000, female SAT scores have been increasing at a rate of 1/8 per year.

Y-intercept: The average female SAT score in 2000 was 498.

(d) The following function shows the relationship between the U.S. consumption of billions of cigarettes, c, and the year since 1960, t

$$c = 7.35t + 484.$$

Slope: Since 1960, cigarette consumption has been increasing at a rate of 7.35 billion cigarettes per year.

Y-intercept: In 1960, 484 billion cigarettes were consumed.

(e) Let the following function represent the relationship between the sales, S, in millions of dollars, for Y years from today,

$$S = 0.8Y + 25.$$

Slope: Sales are projected to increase at a rate of \$800,000 per year.

Y-intercept: This year, sales revenue was \$25 million.

(f) Let the following function represent the relationship between the profit, P, in millions of dollars, for x items sold,

$$P = 1.2x + 250.$$

Slope: For each additional item sold, profit increases by \$1.2 million.

Y-intercept: When zero items are sold, profit is \$250 million.

(g) Let cost, C, in hundreds of dollars, be a linear function of number of computers sold, x

$$C = 0.85x + 12.5$$

Slope: For each computer sold, cost increases by \$85.00.

Y-intercept: If no computers are sold, cost is \$1,250.00.

(h) Let the following function represent the relationship of W, recommended weight in pounds, for a woman f inches over 5 feet tall,

$$W = 100 + 5f$$

Women can weigh however much they want to.

(i) Let the revenue of an Olympic stadium, R, in thousands of dollars, be a function of the number of seats sold, s, be given by

$$R = 205 + .035s$$

Slope: For each additional seat sold, the Revenue at the Olympic stadium increases by \$35.

Y-intercept: When zero seats are sold, the Olympic stadium makes \$205,000.

(j) Let the relationship between the number of payments, P, made and the balance B, in dollars, be given by the following function

$$B = 14,400 - 400P$$

Slope: For each payment made, the balance decreases by \$400.

Y-intercept: The initial balance is \$14,400.

12. Use the following information to write a linear function. (Be careful with what your x and what your y variable is!)

(a) Write loan balance, B, as a linear function of payments, p.

Payments	Loan Balance
2	13,600
4	12,800
6	12,000

-400, -400

$B = -400p + b$ Find b

$12000 = -400(6) + b \Rightarrow b = 14,400$

$\Rightarrow \boxed{B = -400p + 14,400}$

(b) Write salary, S, as a linear function of y, years of employment

Years of employment	Salary
2	15,500
5	20,000
6	21,500
10	27,500

1500, 1500, 1500

$\Rightarrow S = 1500y + b$ Find b

$15500 = 1500(2) + b$

$\Rightarrow b = 12,500$

$\Rightarrow \boxed{S = 1500y + 12,500}$

(c) Write population, p, as a linear function of t, years since 2014

Population	Year
500,000	2014
450,000	2015
350,000	2017

$-50,000$, $-50,000$

$\boxed{p = -50000t + 500,000}$

or $\boxed{p = 500,000 - 50,000t}$

(d) ~~Write recommended weight in lbs, W, for a woman as a linear function of f, inches over 5 feet tall.~~

Total Inches	Weight
63	115
72	160
77	185

(e) Todd had 5 gallons of gasoline in his motorbike. After driving 100 miles, he had 3 gallons left. Write gallons of gasoline as a function of miles. g = gallons. m = miles.

Two points:

$(0,5), (100,3) \Rightarrow \text{slope} = \frac{3-5}{100-0} = -\frac{2}{50} = -\frac{1}{25}$

Point-slope form:

$g - 5 = -\frac{1}{25}(m-0) \Rightarrow \boxed{g = -\frac{1}{25}m + 5}$

(f) You were driving 2 hours, and recorded you traveled 80 miles, and at 4 hours, you recorded your distance as 160 miles. Write distance traveled as a function of time driving, in hours.

Two points: $(2, 80)$ and $(4, 160)$

$\Rightarrow$ Slope $= \frac{160-80}{4-2} = \frac{80}{2} = 40$ mph (Speed)

Y-intercept: $d = 40t + b \Rightarrow 80 = 40(2) + b \Rightarrow \boxed{b=0}$

$\boxed{d = 40t}$ (d: distance, t: time)

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13. The relationship between the balance, B, in dollars, remain on a mortgage loan and the number, n, of monthly payments which you've made is given by the linear function

$$B = 603,200 - 1120n.$$

(This is not how it works in real life)

What is the balance on the mortgage after 10 years? 20 years? 30 years?

10 years: $B = 603,200 - 1120(120) = \boxed{\$468,800}$

20 years: $B = 603,200 - 1120(240) = \boxed{\$334,400}$

30 years: $B = 603,200 - 1120(360) = \boxed{\$200,000}$

Remember, n is measured in months

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14. How would you tell if a function is concave up or concave down with only considering the average rates of change?

If the A.R.C. is increasing over consecutive points, the function is concave up.

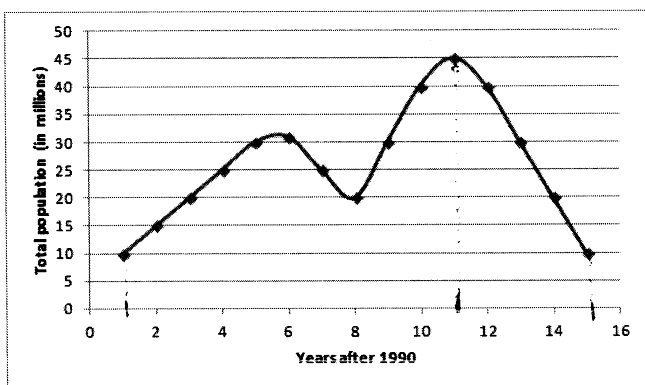
If the A.R.C. is decreasing over consecutive points, the function is concave down.

15. Answer the two questions on page 24 in your supplements book. (This is why math is applicable in 'everyday' life!)

(a) The first description: Thinking of the weight of this individual as a function of time, the weight function is definitely increasing, though it is concave down (increasing at a decreasing rate).

(b) The second description: The rate at which the economy is declining is getting better. This means the function is decreasing, but concave up (decreasing at a less fast rate).

16. Given the following graph, which shows the population of some species in millions with respect to years after 1990. Use this graph to answer the following questions:



(a) Fill in the blanks: Total Population is a function of Years after 1990.

(b) Describe the maximum and where this occurs in the context of the problem.

The maximum is 45 million, and occurs at $t=11$, or in 2001.

(c) Describe the minimum and where this occurs in the context of the problem.

The minimum is 10 million, and occurs in 2 places: at 1991 and 2005.

(d) What years does the population increase? decrease?

In interval notation:

Increasing on $(1, 6)$ and $(8, 11)$

decreasing on $(6, 8)$ and $(11, 15)$

(e) What years is population concave up? concave down?

Concave up: $(7, 9)$

Concave down: $(4, 7)$ and $(9, 13)$,

Everywhere else the function is linear.

(f) What is the domain and range (with units).

Domain: $[1, 15]$ years since 1990

Range: $[10, 45]$ million animals,

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17. Using the data on page 27 in your supplements book answer the following questions:

(a) Write the cost for each of the three plans, ($C_B(x)$, for the cost of the basic plan) where x is the number of extra minutes used.

$$\boxed{\begin{aligned} C_B(x) &= 29.99 + 0.4x \\ C_V(x) &= 39.99 + 0.4x \\ C_P(x) &= 49.99 + 0.4x \end{aligned}}$$

(b) If you were to expect to talk between 9 and 10 hours each month, which would be the best plan for you? Explain.

9 hours = 540 minutes (Basic $x = 240$, value x = Plus $x = 0$)
10 hours = 600 minutes. (Basic $x = 300$, value x = Plus $x = 0$)

For both of these times, the value plan is the cheapest, at \$39.99. The basic costs between \$125.99 and \$149.99, while the Plus plan costs \$49.99.

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18. Fill out the following table, answering the below questions:

Year	Population	Average Rate of Change
1990	4560	NA
1991	4570	10
1992	4581	11
1993	4570	-11
1994	4590	20
1995	5000	410

(a) Between what consecutive years would you use to describe 'the population greatly increasing'? Why? from 1994-1995, since the A.R.C. is greatest for those two years.

(b) Between what consecutive years would you use to describe 'the population greatly decreasing'? Why? from 1992-1993. This is the only negative A.R.C.

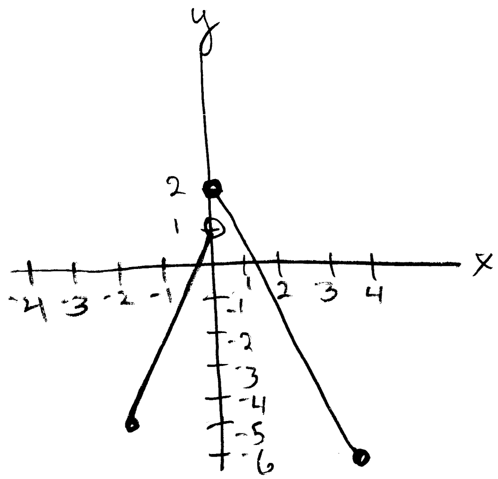
(c) Does the above table represent a linear function, why or why not?

No, because the A.R.C. is not constant between consecutive years.

19. Draw the following piecewise functions:

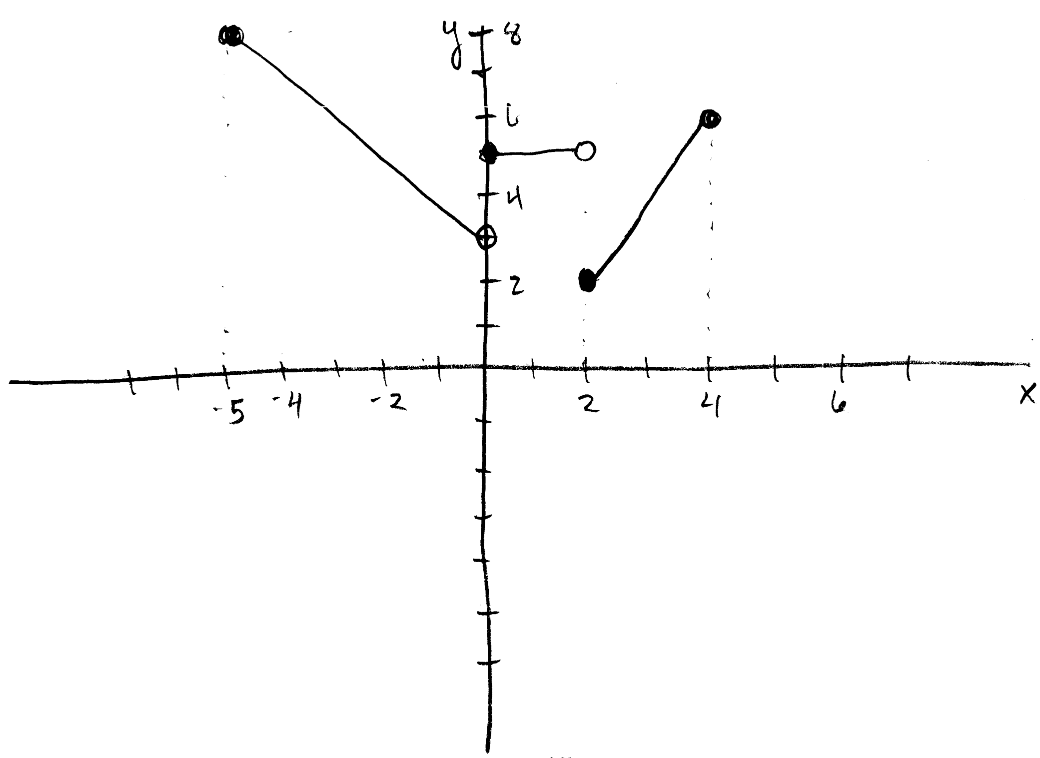
(a)

$$f(x) = \begin{cases} 3x+1 & \text{for } -2 \leq x < 0 \\ 2-2x & \text{for } 0 \leq x \leq 4 \end{cases}$$



(b)

$$g(x) = \begin{cases} 3-x & \text{for } -5 \leq x < 0 \\ 5 & \text{for } 0 \leq x < 2 \\ 2x-2 & \text{for } 2 \leq x \leq 4 \end{cases}$$

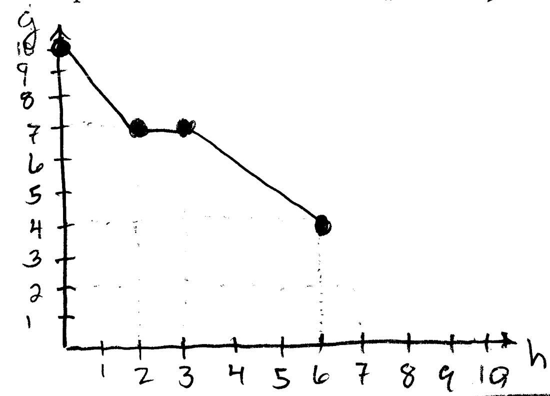


20. If you were on a road trip, and started with 10 gallons of gas in your car. You drove for 2 hours, and ended up with 7 gallons of gas in your car. You stopped for an hour lunch, then drove for 3 more hours and ended up with 4 gallons of gas in your car.

(a) Draw a piecewise function to represent ~~hours~~ gallons as a function of ~~gallons~~ hours of gas in your car.

hours = h

gallons of gas = g



$g = -h + b$ (To find b, plug in $g = 7$, $h = 3$)

$7 = -3 + b \Rightarrow b = 10$

(b) Write the ~~piecewise function~~.

$$\boxed{g = \begin{cases} -\frac{3}{2}h + 10 & \text{for } 0 \le h < 2 \\ 7 & \text{for } 2 \le h < 3 \\ -h + 10 & \text{for } 3 \le h \le 6 \end{cases}}$$

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21. The table below gives the number of calculators produced, p, and the total dollar cost, c.

p	c
200	16,320
400	23,048
600	30,150
800	37,616
1000	45,440
1200	53,640

(a) Write the best fit linear function (round to 4 decimals), to represent cost as a linear function of number of calculators produced.

Linear regression;

$$\boxed{c = 37.3203p + 8244.8}$$

(b.1) What is the correlation coefficient (round to 4 decimals)? And what two things does this tell us?

$\boxed{r = 0.9994}$

(b.2) Does the correlation coefficient say that an increase in p causes an increase in c? Why or why not?

Yes, because the correlation coefficient is positive, so there is a positive correlation between p and c.

(c) State the two numerical values in part (a) and explain what they represent in the context of the problem.

$a = 37.3203$ (Slope)

For each additional calculator produced, the cost increases by about \$37.32.

$b = \$8{,}244.80$. The cost to produce zero calculators (overhead) is about \$8,244.80.

(d) How much would producing 100 calculators cost in dollars? Is this extrapolation or interpolation, explain (be careful!)

$$c = 37.3203(100) + 8244.8$$
$$= \$11{,}976.83$$

$\boxed{\text{Extrapolation}}$

(e) How much would producing 900 calculators cost in dollars? Is this extrapolation or interpolation, explain (be careful!)

$$c = 37.3203(900) + 8244.8$$
$$= \boxed{\$41{,}833.07}$$

$\boxed{\text{Interpolation}}$

(f) How much would producing 1000 calculators cost in dollars, according to the best fit linear function? Does this match the data? What does result say, if anything, about our data? our function?

$$C = 37.3203(1000) + 8244.8$$
$$= \boxed{\$45,565.10}$$

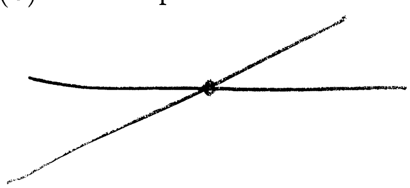
The estimated value is very close to the actual value, but not equal. This says that our line is a very good fit, but simply that the data is not precisely linear.

22. Answer the following questions:

(a) If two lines have no solution, what does this say about the two lines?

They are parallel

(b) Draw a picture of two lines that have one solution.



(c) What if two lines have infinitely many solutions, describe the system?

The two lines are the same

23. Jack is producing and selling T-shirts. His manager found that the demand equation is given by $3p + q = 303$ and the supply equation is given by $p = 9 + 0.05q$ (where q is the quantity of *hundreds of* T-shirts and p is price in dollars).

(a) What is the equilibrium price?

$$3p + q = 303$$
$$p = 9 + 0.05q$$
$$3(9 + 0.05q) + q = 303$$
$$27 + 0.15q + q = 303$$
$$1.15q = 276$$
$$\boxed{q = 240}$$

$$p = 9 + 0.05(240)$$
$$= 9 + 12$$
$$= \boxed{21}$$

equilibrium price = \$21.00

(b) What is the equilibrium quantity (be very careful with this)?

equilibrium quantity

(c) If price was fixed at \$15, what would occur (i.e. shortage or surplus), *and why*? (Note, I must see work to support your answer and reason, either mathematically or graphically.)

\$15 is less than the equilibrium price, so supply will be less than demand (shortage):

Supply: $15 = 9 + 0.05q \Rightarrow 0.05q = 6 \Rightarrow q = 120$

Demand: $3(15) + q = 303 \Rightarrow q = 303 - 45 \Rightarrow q = 258$

$\boxed{\text{So supply is 120, while demand is 258}}$

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24. Assume you have \$2000 to invest for one year. You can make a safe investment that yields 4% interest a year or a risky investment that yields 9% a year.

(a) How much interest will you earn in ~~one year~~ if you invest all of your money in the safe investment?

$(0.04)(\$2000) = \boxed{\$80}$

(b) How much interest will you earn in one year if you invest all of your money in the risky investment?

$(0.09)(\$2000) = \boxed{\$180}$

(c) Suppose you invest x dollars in the safe investment and y dollars in the risky investment. How much interest will you earn in one year? (Note you will not have an actual numerical value as in parts (a) and (b).)

$\boxed{0.04x + 0.09y}$

(d) Suppose you still have a total of \$2000, and also want to combine safe and risky investments to earn \$100 a year. (1) Find a system of two linear equations that x and y must satisfy (note, you should have *part of* one of these already!). How (2) much should you invest in the risky, and (3) how much in the safe? Solve using any algebraic method you would like (i.e. not graphically!).

$$0.04x + 0.09y = 100$$
$$x + y = 2000$$
$$x = 2000 - y$$
$$\Rightarrow 0.04(2000 - y) + 0.09y = 100$$
$$\Rightarrow 80 - 0.04y + 0.09y = 100$$
$$\Rightarrow 0.05y = 20$$
$$\Rightarrow \boxed{y = 400} \Rightarrow \boxed{x = 1600}$$

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25. By substituting specific values of a, m and n show that, in general, $a^n + a^m$ is not equal to a^{n+m}.

$a = 2$ $\qquad$ $a^n + a^m = 4 + 8 = 12$

$m = 2$ $\qquad$ $a^{n+m} = 2^5 = 32$

$n = 3$

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26. Why does $(a+b)^2 \neq a^2 + b^2$ in general?

(a) Find a specific case for values of a and b where these expressions are not equal.

$a =$ $\qquad$ $(a+b)^2 =$

$b =$ $\qquad$ $a^2 + b^2 =$

(b) Expand $(a+b)^2 = (a+b)(a+b)$ and use the expanded expression to determine when $(a+b)^2$ will equal $a^2 + b^2$.

$$(a+b)^2 = a^2 + 2ab + b^2 \Rightarrow (a+b)^2 = a^2 + b^2$$
$$\Leftrightarrow 2ab = 0$$
$$\Leftrightarrow \boxed{a = 0 \text{ or } b = 0}$$

27. Simplify and express your answer using only positive exponents: $\dfrac{(a^3bc)^5}{(ab^{-2})^{-4}\,c^2}$

$= \dfrac{a^{15}b^5c^5}{a^{-4}b^{-8}c^2} = \boxed{a^{19}b^{13}c^3}$

28. Simplify the following expression, express your answer using only positive exponents. $\left(\frac{a}{b}\right)^3 \cdot \left(\frac{a}{b}\right)^{-5}$

$\left(\frac{a}{b}\right)^3 \cdot \left(\frac{a}{b}\right)^{-5} = \left(\frac{a}{b}\right)^{3-5} = \left(\frac{a}{b}\right)^{-2} = \boxed{\left(\frac{b}{a}\right)^2}$

29. By substituting specific values of a and b, show that, in general, $\sqrt{a+b}$ is not equal to $\sqrt{a}+\sqrt{b}$ and $\sqrt[3]{a+b}$ is not equal to $\sqrt[3]{a}+\sqrt[3]{b}$.

$a = 9$

$b = 16$

$$\begin{aligned}
\sqrt{a+b} &= \sqrt{25} = 5 \\
\sqrt{a}+\sqrt{b} &= 3+4 = 7 \\
\sqrt[3]{a+b} &= \sqrt[3]{25} \approx 2.9240 \\
\sqrt[3]{a}+\sqrt[3]{b} &= \sqrt[3]{9}+\sqrt[3]{16} \approx 4.5999
\end{aligned}$$

30. Calculate the following (note, the calculator may not always give the complete answer!):

(a) $625^{1/4}$ 5

(b) $(-625)^{1/4}$ Imaginacy

(c) $125^{1/3}$ 5

(d) $(-125)^{1/3}$ -5

31. Simplify the following expressions, (assume all variables are nonnegative real numbers)

(a) $\sqrt[3]{625x^4}$ $= \sqrt[3]{5^4 \cdot x^4} = \boxed{5^{4/3}\, x^{4/3}}$

(b) $3\sqrt{48} - 5\sqrt{27}$

$3\sqrt{16\cdot 3} - 5\sqrt{9\cdot 3}$

$= 3\cdot 4\sqrt{3} - 5\cdot 3\sqrt{3}$

$= 12\sqrt{3} - 15\sqrt{3} = \boxed{-3\sqrt{3}}$

32. Rewrite $\sqrt[25]{x^7}$ as a fractional power. $x^{7/25}$

33.

(a) Write 2,300,000,000,000 in scientific notation. $\boxed{2.3\times 10^{12}}$

(b) Write 0.000000000000023 in scientific notation. $\boxed{2.3\times 10^{-14}}$

(c) Write 6.5×10^{-9} in standard notation. $\boxed{0.0000000065}$

(d) How many zeros does 6.5×10^{90} have? $\boxed{89}$