

**Review:**

These document provides you a list of questions that you should be able to complete by the end of the semester. You should also be able to define any terms on the ‘Key Ideas’ sheet also provided for you. (Some terms may not be tested on this sheet.)

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1. Using the data on page 15 of your Supplements book (Countries from Algeria to Haiti... *not* onto page 16) efficiently turn this data into a frequency and relative frequency table, given below:

Life Expectancy	Frequency of Countries	Relative Frequency of Countries
30 – 39		
40 – 49		
50 – 59		
60 – 69		
70 – 79		
80 – 89		

Draw two histograms to represent this data (one with frequency, one with relative frequency):

Explain in your own words what is the ‘best’ way (what does ‘best’ mean?) to represent this data, table given in Supplements book, table you provided above or histogram? And why is this the ‘best’? (There is no ‘right’ answer, but make sure your reasoning supports your answer—this could be wrong.)

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2. Below is a list of some winter Olympic sports, and the number of people surveyed who favor the sport.

Sport	Number of People
Alpine Skiing	100
Biathlon	200
Curling	153
Ice Hockey	255
Speed Skating	96

- (a) What is the frequency and relative frequency of the number of people who favor curling? Speed skating?
- (b) Can you state which sport has the highest relative frequency, just given the list of ‘number of people’ and without calculations? Why or why not?

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3. Answer the following questions concerning different types of data:

(a) If you have data that is skewed left, which will be greater, mean or median and why?

(b) If you have data that is skewed right, which will be greater, mean or median and why?

(c) Draw an example of a bar chart of data that is unimodal.

(d) Draw an example of a bar chart of data that is skewed right.

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4. Using the data on page 9 in the Supplements book (page is titled ‘The Mean Truth about Means’) answer the following questions:

(a) Using the top 10 countries, rounding to the nearest million (i.e. China would be 1,339,000,000 not 1,338,612,968) find the mean. It is fine to not write all mathematical work, but explain the process you used in order to state the mean.

(b) State the median. Again, explain the process you used in order to solve this problem.

(c) Why would one use the mean?

(d) Why would one use the median?

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5. I'm training for a race, and have ran a few days. My mileage, for the past 5 runs has been

2.19, 2.30, 4.10, 2.32, 2.25

If I wanted to look as impressive as I could, would I use the mean or median to describe my 'average' runs? Why?

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6. Answer the following questions: Given these four test results. The numbers given are the number of problems correct, out of 25

18, 15, 22, 20.

(a) What is the mean?

(b) What is the median?

- (c) Would you rather state your mean or median as your average and why?
- (d) If you wanted a mean of 80% on five exams what would you want the number correct to be (out of 25 questions) for your fifth exam?
- (e) If you wanted a median of 80% on five exams what would you want the number correct to be (out of 25 questions) for your fifth exam? (List all possible, if there is more than one possibility)
- (f) With any number of tests, is it ever possible to have a mean of exactly 100%? Why or why not?
- (g) With any number of tests, is it ever possible to have a median of exactly 100%? Why or why not?

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7. Know how to tell if a point is a solution of a function (given the function, or graph of function). This can get tricky, if you are asked to find  $f(5)$ , what is 5? The  $x$ -value or  $y$ -value? Homework has lots of examples of this, if you do not understand, go through these examples, and/or come to my office to talk about it.

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8. If you were to be asked to write  $h$ , height, as a function of  $t$ , time, what is the  $x$ - and what is the  $y$ - value?

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9. If the height of an animal increases by 6 inches, every year, write height,  $h$ , as a function of years,  $t$ , when the animal was born with a height of 2 inches.

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10. Given the below scenarios state if the description is a function and if so state the input and output of these, if not state why not a function:

(a) The total snowfall for each day on a given month.

(b) The highest exam score for each student in this class.

(c) The exam scores for each student in this class at the end of the year.

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11. Given the following linear functions and descriptions. State the units and what the slope and  $y$ -intercept represent in the context of the problem.

- (a) The relationship between the balance,  $B$ , in dollars, remain on a mortgage loan and the number,  $n$ , of monthly payments which you've made is given by the linear function

$$B = 403,200 - 1120n.$$

- (b) Annual Income in dollars,  $d$ , is a function of members of household,  $n$ , given by the function

$$d = 6438n + 12451$$

- (c) The average female math SAT scores,  $m$ , is a function of years since 2000,  $t$ , and can be represented as

$$m = \frac{1}{8}t + 498.$$

- (d) The following function shows the relationship between the U.S. consumption of billions of cigarettes,  $c$ , and the year since 1960,  $t$

$$c = 7.35t + 484.$$

- (e) Let the following function represent the relationship between the sales,  $S$ , in millions of dollars, for  $Y$  years from today,

$$S = 0.8Y + 25.$$

- (f) Let the following function represent the relationship between the profit,  $P$ , in millions of dollars, for  $x$  items sold,

$$P = 1.2x + 250.$$

- (g) Let cost,  $C$ , in hundreds of dollars, be a linear function of number of computers sold,  $x$

$$C = 0.85x + 12.5$$

- (h) Let the following function represent the relationship of  $W$ , recommended weight in pounds, for a woman  $f$  inches over 5 feet tall,

$$W = 100 + 5f$$

- (i) Let the revenue of an Olympic stadium,  $R$ , in thousands of dollars, be a function of the number of seats sold,  $s$ , be given by

$$R = 205 + .035s$$

- (j) Let the relationship between the number of payments,  $P$ , made and the balance  $B$ , in dollars, be given by the following function

$$B = 14,400 - 400P$$

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12. Use the following information to write a linear function. (Be careful with what your  $x$  and what your  $y$  variable is!)

- (a) Write loan balance,  $B$ , as a linear function of payments,  $p$ .

Payments	Loan Balance
2	13,600
4	12,800
6	12,000

- (b) Write salary,  $S$ , as a linear function of  $y$ , years of employment

Years of employment	Salary
2	15,500
5	20,000
6	21,500
10	27,500

- (c) Write population,  $p$ , as a linear function of  $t$ , years since 2014

Population	Year
500,000	2014
450,000	2015
350,000	2017

- (d) Write recommended weight in lbs,  $W$ , for a woman as a linear function of  $f$ , inches over 5 feet tall.

Total Inches	Weight
63	115
72	160
77	185

- (e) Todd had 5 gallons of gasoline in his motorbike. After driving 100 miles, he had 3 gallons left. Write gallons of gasoline as a function of miles.

- (f) You were driving 2 hours, and recorded you traveled 80 miles, and at 4 hours, you recorded your distance as 160 miles. Write distance traveled as a function of time driving, in hours.

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13. The relationship between the balance,  $B$ , in dollars, remain on a mortgage loan and the number,  $n$ , of monthly payments which you've made is given by the linear function

$$B = 603,200 - 1120n.$$

What is the balance on the mortgage after 10 years? 20 years? 30 years?

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14. How would you tell if a function is concave up or concave down with only considering the average rates of change?

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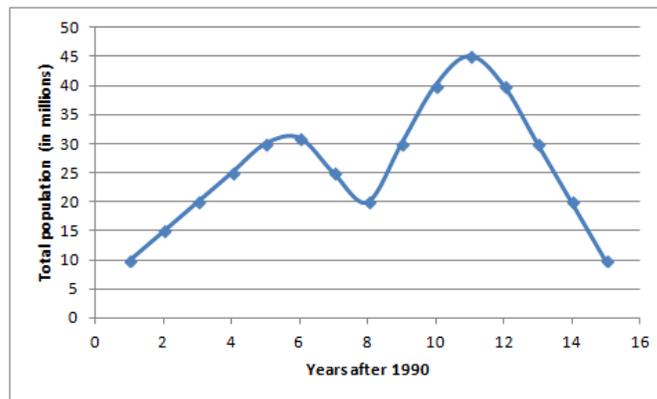
15. Answer the two questions on page 24 in your supplements book. (This is why math is applicable in 'everyday' life!)

(a) The first description:

(b) The second description:

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16. Given the following graph, which shows the population of some species in millions with respect to years after 1990. Use this graph to answer the following questions:



(a) Fill in the blanks: \_\_\_\_\_ is a function of \_\_\_\_\_.

(b) Describe the maximum and where this occurs in the context of the problem.

(c) Describe the minimum and where this occurs in the context of the problem.

(d) What years does the population increase? decrease?

(e) What years is population concave up? concave down?

(f) What is the domain and range (with units).

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17. Using the data on page 27 in your supplements book answer the following questions:

(a) Write the cost for each of the three plans, ( $C_B(x)$ , for the cost of the basic plan) where  $x$  is the number of extra minutes used.

- (b) If you were to expect to talk between 9 and 10 hours each month, which would be the best plan for you? Explain.

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18. Fill out the following table, answering the below questions:

Year	Population	Average Rate of Change
1990	4560	
1991	4570	
1992	4581	
1993	4570	
1994	4590	
1995	5000	

- (a) Between what consecutive years would you use to describe ‘the population greatly increasing’? Why?
- (b) Between what consecutive years would you use to describe ‘the population greatly decreasing’? Why?
- (c) Does the above table represent a linear function, why or why not?

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19. Draw the following piecewise functions:

(a)

$$f(x) = \begin{cases} 3x + 1 & \text{for } -2 \leq x < 0 \\ 2 - 2x & \text{for } 0 \leq x \leq 4 \end{cases}$$

(b)

$$g(x) = \begin{cases} 3 - x & \text{for } -5 \leq x < 0 \\ 5 & \text{for } 0 \leq x < 2 \\ 2x - 2 & \text{for } 2 \leq x \leq 4 \end{cases}$$

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20. If you were on a road trip, and started with 10 gallons of gas in your car. You drove for 2 hours, and ended up with 7 gallons of gas in your car. You stopped for an hour lunch, then drove for 3 more hours and ended up with 4 gallons of gas in your car.

(a) Draw a piecewise function to represent hours as a function of gallons of gas in your car.

(b) Write the piecewise function.

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21. The table below gives the number of calculators produced,  $p$ , and the total dollar cost,  $c$ .

$p$	$c$
200	16,320
400	23,048
600	30,150
800	37,616
1000	45,440
1200	53,640

(a) Write the best fit linear function (round to 4 decimals), to represent cost as a linear function of number of calculators produced.

- (b.1) What is the correlation coefficient (round to 4 decimals)? And what two things does this tell us?
- (b.2) Does the correlation coefficient say that an increase in  $p$  causes an increase in  $c$ ? Why or why not?
- (c) State the two numerical values in part (a) and explain what they represent in the context of the problem.
- (d) How much would producing 100 calculators cost in dollars? Is this extrapolation or interpolation, explain (be careful!)
- (e) How much would producing 900 calculators cost in dollars? Is this extrapolation or interpolation, explain (be careful!)

- (f) How much would producing 1000 calculators cost in dollars, according to the best fit linear function? Does this match the data? What does result say, if anything, about our data? our function?

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22. Answer the following questions:

- (a) If two lines have no solution, what does this say about the two lines?
- (b) Draw a picture of two lines that have one solution.
- (c) What if two lines have infinitely many solutions, describe the system?

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23. Jack is producing and selling T-shirts. His manager found that the demand equation is given by  $3p + q = 303$  and the supply equation is given by  $p = 9 + 0.05q$  (where  $q$  is the quantity of *hundreds* of T-shirts and  $p$  is price in dollars).

- (a) What is the equilibrium price?

(b) What is the equilibrium quantity (be very careful with this)?

(c) If price was fixed at \$15, what would occur (i.e. shortage or surplus), *and why?* (Note, I must see work to support your answer and reason, either mathematically or graphically.)

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24. Assume you have \$2000 to invest for one year. You can make a safe investment that yields 4% interest a year or a risky investment that yields 9% a year.

(a) How much interest will you earn in one year if you invest all of your money in the safe investment?

(b) How much interest will you earn in one year if you invest all of your money in the risky investment?

(c) Suppose you invest  $x$  dollars in the safe investment and  $y$  dollars in the risky investment. How much interest will you earn in one year? (Note you will not have an actual numerical value as in parts (a) and (b).)

- (d) Suppose you still have a total of \$2000, and also want to combine safe and risky investments to earn \$100 a year. (1) Find a system of two linear equations that  $x$  and  $y$  must satisfy (note, you should have *part of* one of these already!). How (2) much should you invest in the risky, and (3) how much in the safe? Solve using any algebraic method you would like (i.e. not graphically!).

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25. By substituting specific values of  $a$ ,  $m$  and  $n$  show that, in general,  $a^n + a^m$  is not equal to  $a^{n+m}$ .

$$\begin{array}{ll}
 a = & a^n + a^m = \\
 m = & a^{n+m} = \\
 n = &
 \end{array}$$

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26. Why does  $(a + b)^2 \neq a^2 + b^2$  in general?

- (a) Find a specific case for values of  $a$  and  $b$  where these expressions are not equal.

$$\begin{array}{ll}
 a = & (a + b)^2 = \\
 b = & a^2 + b^2 =
 \end{array}$$

- (b) Expand  $(a + b)^2 = (a + b)(a + b)$  and use the expanded expression to determine when  $(a + b)^2$  will equal  $a^2 + b^2$ .

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27. Simplify and express your answer using only positive exponents:  $\frac{(a^3bc)^5}{(ab^{-2})^{-4}c^2}$

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28. Simplify the following expression, express your answer using only positive exponents.  $\left(\frac{a}{b}\right)^3 \cdot \left(\frac{a}{b}\right)^{-5}$

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29. By substituting specific values of  $a$  and  $b$ , show that, in general,  $\sqrt{a+b}$  is not equal to  $\sqrt{a} + \sqrt{b}$  and  $\sqrt[3]{a+b}$  is not equal to  $\sqrt[3]{a} + \sqrt[3]{b}$ .

$a =$

$$\sqrt{a+b} =$$

$$\sqrt{a} + \sqrt{b} =$$

$b =$

$$\sqrt[3]{a+b} =$$

$$\sqrt[3]{a} + \sqrt[3]{b} =$$

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30. Calculate the following (note, the calculator may not always give the complete answer!):

(a)  $625^{1/4}$

(b)  $(-625)^{1/4}$

(c)  $125^{1/3}$

(d)  $(-125)^{1/3}$

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31. Simplify the following expressions, (assume all variables are nonnegative real numbers)

(a)  $\sqrt[3]{625x^4}$

(b)  $3\sqrt{48} - 5\sqrt{27}$

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32. Rewrite  $\sqrt[25]{x^7}$  as a fractional power.

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33.

(a) Write 2,300,000,000,000 in scientific notation.

(b) Write 0.0000000000000023 in scientific notation.

(c) Write  $6.5 \times 10^{-9}$  in standard notation.

(d) How many zeros does  $6.5 \times 10^{90}$  have?

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34. By completing this question you should have a good idea of the differences of Linear Functions and Exponential Functions.

- (a) What does a basic linear function look like?
  
- (b) What does a basic exponential function look like?
  
- (c) If my function  $f(t) = 150 + 50t$  models  $f(t)$  the turtle population at  $t$  years. What does 150 represent? What does 50 represent? (in the context of the problem)
  
  
  
  
  
  
  
  
  
  
- (d) If my function  $f(t) = 150(1.29)^t$  models  $f(t)$  the turtle population at  $t$  years. What does 150 represent? What does 1.29 represent? (in the context of the problem)
  
  
  
  
  
  
  
  
  
  
- (e) If you were given a table of  $x$  and  $y$  values, how could you tell if the table represented a linear function?
  
  
  
  
  
  
  
  
  
  
- (f) If you were given a table of  $x$  and  $y$  values, how could you tell if the table represented an exponential function?
  
  
  
  
  
  
  
  
  
  
- (g) Fill in the blank: A linear function represents a quantity to which a constant amount is \_\_\_\_\_ for each unit increase in the input.
  
- (h) Fill in the blank: An exponential function represents a quantity that is \_\_\_\_\_ by a constant factor for each unit increase in the input.

- (i) What does the average rate of change do within a linear function?
- (j) What does the average rate of change do within an exponential function?
- (k) If you were given a linear function  $y = mx + b$ . Can  $m$  be negative? If so, what does this tell us?
- (l) If you were given a linear function  $y = mx + b$ . Can  $m$  be positive? If so, what does this tell us?
- (m) If you were given a linear function  $y = mx + b$ . Can  $m$  be 1? If so, what does this tell us?
- (n) If you were given an exponential function  $y = Ca^x$  can  $a$  be negative? If so, what does this tell us?
- (o) If you were given an exponential function  $y = Ca^x$  can  $a$  be positive? If so, what does this tell us? (Hint: you need to separate into two cases,  $a > 1$  and  $0 < a < 1$ )
- (p) If you were given an exponential function  $y = Ca^x$  can  $a$  be 1? If so, what does this tell us?
- (q) If you have an exponential growth function,  $y = Ca^x$ , what must be the restriction?
- (r) If you have an exponential decay function,  $y = Ca^x$ , what must be the restriction?

(s) If you have an exponential growth function,  $y = Ca^x$ , what is true when  $x \rightarrow -\infty$ ? And what does this say graphically?

(t) If you have an exponential decay function,  $y = Ca^x$ , what is true when  $x \rightarrow \infty$ ? And what does this say graphically?

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35. You are studying the population of red-eared slider turtles. You have come up with the following equation to represent the turtle population  $t$  years after you initially started studying their population.

$$p(t) = 120(1.4)^t.$$

(a) What does 120 mean in the context of the problem?

(b) What does 1.4 mean in the context of the problem?

(c) What is the growth/decay rate and what does this mean in the context of the problem?

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36. Answer the following questions

- (a) A population of something grows exponentially, in 2002 the population was 120 million, in 2010 the population was 340 million. Write the function,  $P(t)$ , population in millions where  $t$  is years since 2000 that represents this data.

- (b) A population has a doubling time of 10 years. If the population started at 5, write the function  $P(y)$  where  $y$  is one year. Write the function  $P(d)$  where  $d$  is in decades. Write the function  $P(m)$  where  $m$  is in months. (Do exact and round growth (or decay) factor to 4 decimals.)

(c) A substance decays exponentially. At the 5th hour there is 500 mg at the 10th hour there is 300 mg. Write the function  $S(h)$ , mg of the substance, where  $h$  is in hours.

(d) Something is decaying exponentially. At the start of the study there are 15 units, 10 hours after there are 2.3 units. Write the function  $U(h)$ , number of units, for  $h$  hours.

(e) The profit of a company grows exponentially. The rate of growth is 5% per quarter. If this year's profit started at \$5000, write the profit of the company,  $P(q)$  where  $q$  is quarter. Write the profit of the company  $P(y)$  where  $y$  is years.

(f) If a substance decays by 10% each hour, and the initial substance is 600 units. Write the function  $F(t)$  where  $t$  is in days.

(g) You come up with the function

$$P(t) = 100(1.05)^t$$

where  $P(t)$  is the population  $t$  years since 2000. Describe what 100 and 1.05 mean in the context of the problem. And state the growth rate, and explain what this means in the context of the problem.

(h) You come up with the function

$$P(t) = 100(0.95)^t$$

where  $P(t)$  is the population  $t$  years since 2000. Describe what 100 and 0.95 mean in the context of the problem. And state the decay rate, and explain what this means in the context of the problem.

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37. Answer the following questions, by writing a function to describe the situation.
- (a) A drug is eliminated from the body at a rate of 25% hourly. Write a function in terms of  $h$ , hours, if there was an initial amount of 40 mg.
  
  
  
  
  
  
  
  
  
  
  - (b) A drug is eliminated from the body at a rate of 25% continuously. Write a function in terms of  $h$ , hours, if there was an initial amount of 40 mg.
  
  
  
  
  
  
  
  
  
  
  - (c) A drug is eliminated from the body at a rate of 25% hourly. Write a function in terms of  $h$ , hours, if there was an initial amount of 400 mg.
  
  
  
  
  
  
  
  
  
  
  - (d) A drug has a half-life of 5 hours. Write a function in terms of  $h$ , hours, if there is an initial value of 60 mg.
  
  
  
  
  
  
  
  
  
  
  - (e) Write the above function as a continuous decay function.
  
  
  
  
  
  
  
  
  
  
  - (f) If you have the exponential growth function of  $500e^{0.456t}$ , write this in the form  $Ca^t$ .
  
  
  
  
  
  
  
  
  
  
  - (g) A population is increasing at a rate of 12% every 5 years. Write a function in terms of  $t$ , years, if there is an initial population of 400.

(h) A population is increasing at a continuous rate of 12%. Write a function in terms of  $t$ , years, if there is an initial population of 400.

(i) A population has a doubling time of 50 years. Write a function in terms of  $t$ , years, if there is an initial population of 100.

(j) Write the above function as a continuous growth function.

(k) If you have an exponential decay function of  $500e^{-0.456t}$ , write this in the form  $Ca^t$ .

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38. The two formulas are the only two that will be given on the exam:

$$A = P \left(1 + \frac{r}{n}\right)^{nt} \qquad A = Pe^{rt}$$

For example,

(a) If you invest \$100 in an account earning 5% compounded semi-annually. How much will you have at the end of 4 years?

- (b) If you invest \$100 in an account earning 5% compounded continuously. How much will you have at the end of 4 years?
- (c) If you want to have \$1000 at the end of 4 years, and you put this into an account earning 5% compounded semi-annually. How much should you invest at the start of 4 years?
- (d) If you want to have \$1000 at the end of 4 years, and you put this into an account earning 5% compounded continuously. How much should you invest at the start of 4 years?
- (e) If you want to have \$1000 at the end of 10 years, and you start with \$800. How much should your interest rate be, if it is compounded quarterly?

(f) How much should you start off with in an account that earns interest at 2.2% compounded daily, and you want \$200 more dollars in 10 years? (can be tricky, write everything that you know, and figure out what you need to solve for. Say, you start off with  $P$  dollars and end up with  $P + 200$  dollars.)

(g) What is the effective interest rate if you invest in an account that earns 6.5% daily? continuously?

(h) What is the annual and semi annual nominal interest rate if the effective interest rate is 5.7%?

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39. Using the rule of 70 answer the following questions:

(a) If a population has a growth rate of 5% per year, will this have a doubling time or half-life, explain? Approximate this.

(b) If a population has a decay rate of 5% per year, will this have a doubling time or half-life, explain? Approximate this.

(c) According to this function

$$P(t) = 500(1.20)^t$$

where  $t$  is in years, will this function have a doubling time or half-life, explain? Approximate this.

(d) According to this function

$$P(t) = 500(0.20)^t$$

where  $t$  is in days, will this function have a doubling time or half-life, explain? Approximate this.

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40. The following table gives the number of U.S. jobs supported by exports to Mexico for recent years and can be found in Glassman. Number is in thousands. For your regression function let  $t = 0$  correspond to year 1986. (With this information you will need to adjust your table prior to inputting in calculator.)

Year	Jobs
1986	274
1987	300
1988	400
1989	500
1990	590
1991	700
1992	900

- (a) Write the best fit function (either linear or exponential), round to 4 decimals.
- (b) What is the correlation coefficient (round to 4 decimals)? And what two things does this tell us?
- (c) State the two numerical values in part (a) and explain what they represent in the context of the problem.

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41. Consider the following table which describes a cat population  $t$  years after 2000:

Year ( $t$ )	Cat Population
0	100
2	200
4	380
6	753
8	1477
10	2895

- (a) What is the best fit exponential function? (round to 4 decimals).
- (b) Why is the initial value in the regression function 100.4921 not the same as the table's initial value, 100? What does this say about our regression function?
- (c) What is the correlation coefficient (round to 5 decimals)? Does this say our *data* is accurate? If not, what does this indicate is accurate?
- (d) In our above function, is  $t$  every year, or every 2?
- (e) What is the growth rate? What does this tell us?

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42. True or False. (If False know what was done wrong.) Note, the same true properties hold for  $\ln$ . I'm not rewriting these problems, but we could rewrite as  $\ln$ .

(a)  $\log_{10} x = 10^x$

(i)  $\log x^2 = (\log x)^2$

(b)  $\log_{10}(x + 1) = \log(x + 1)$

(j)  $\log\left(\frac{x}{y}\right) = \frac{\log x}{\log y}$

(c)  $\log(x + y) = \log x + \log y$

(k)  $\log x - \log y = \frac{\log x}{\log y}$

(d)  $\log(x + y) = \log x + y$

(l)  $\log x - \log y = \log\left(\frac{x}{y}\right)$

(e)  $\log(x + y) = \log x \cdot \log y$

(m)  $\log x - \log y = \frac{\log x}{y}$

(f)  $3 \log x = \log x^3$

(n)  $\log(xy) = \log x \cdot \log y$

(g)  $2 \log(x + y) = \log(x^2 + y^2)$

(h)  $\log x^2 = \log x + \log x$

(o)  $\log(x^2 + 4x + 4) = 2 \log(x + 2)$

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43. Solve the following equations for the variable. Make sure you can solve *exactly* and solve rounding to the proper decimal. (Note, some may not exist or be messy. Note, you cannot take the natural log, or log of a negative number.)

(a)  $6^x - 10^{3x+2} = 0$

(b)  $10^{x^2} = 100^{x^2-6}$

$$(c) e^{x+8} = 10^x$$

$$(d) 1005 = 123e^x$$

$$(e) 1005 = 123(x)^4$$

$$(f) 8^x - 2^{x+6} = 0$$

$$(g) 5e^{2x+5} = 100$$

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44. If possible expand using the properties of logarithms, if not possible state why.

(a)  $\ln(x^3\sqrt{yz})$

(b)  $\ln\left(\frac{x^3}{\sqrt{yz}}\right)$

(c)  $\ln\left(\frac{x^3}{\sqrt{y}}\sqrt{z}\right)$

(d)  $5\ln(x+y)$

(e)  $5\ln\left(\sqrt{x}\cdot\frac{z}{\sqrt[3]{y}}\right)$

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45. Contract, rewriting the expression as a single logarithm.

(a)  $\frac{1}{2} \ln x - \frac{3}{2} \ln(x + 1)$

(b)  $3 \ln(x) + 5 \ln(y) - \ln(z)$

(c)  $4(\ln x + \ln y - \ln z)$

(d)  $\ln(z + 5) - \ln(z - 5)$

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46. Answer the following financial problems.

- (a) If you want to have \$1000 at the end of 10 years, and you start with \$800. How much should your interest rate be, if it is compounded continuously?

(b) How many years will it take \$500 to grow to \$600 with an interest rate of 3% compounded monthly?

(c) How many years will it take \$500 to grow to \$600 with an interest rate of 3% compounded continuously? (Round to the whole year)

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47. Let  $f(t)$  be the quantity of ampicillin, in mg, in the bloodstream at time  $t$  hours since the drug was taken. At  $t = 0$ , the amount is 250 mg, and approximately 42% of the drug is eliminated each hour.

(a) Write the exponential function,  $f(t)$  for the amount of drug in the system.

(b) Find exactly how many hours (rounding to two decimals) it will take for there to be 100 mg of the drug in the bloodstream. Showing all algebraic work.

- (c) What if the drug was continuously eliminated at a rate of 42%? What is the exponential function then?
- (d) How many hours (rounding to two decimals) will it take for there to be 100 mg of the drug in the bloodstream for your function in part (c). Show all algebraic work.

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48. Answer the following questions (not using the rule of 70 but solving exactly).

- (a) If a population has a growth rate of 5% per year, will this have a doubling time or half-life? Explain and find this.
- (b) If a population has a decay rate of 5% per month, will this have a doubling time or half-life? Explain and find this.
- (c) According to this function  $P(t) = 500(1.20)^t$  where  $t$  is in days. Will this function have a doubling time or half-life? Explain and find this.

- (d) According to this function  $P(t) = 500(0.20)^t$  where  $t$  is in hours. Will this function have a doubling time or half-life. Explain and find this.
- (e) Find the exact doubling time for a substance that earns increases 5% per quarter year. (Round to the nearest quarter year.)
- (f) Find the exact half-life for a radioactive substance that has an annual decay rate of 2.4%. (Round to the nearest year.)

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49. Answer the following questions about logarithms.

- (a) What is the domain of  $\log x$ ?
- (b) What is the range of  $\log x$ ?
- (c) Is there a horizontal asymptote? Where is this located?
- (d) Is there a vertical asymptote? Where is this located?
- (e) What is the difference between  $\log x$  and  $\ln x$ ?

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50. Solve the following exactly. (Keep in mind the domain of log and ln.)

(f)  $\ln(x) - \ln(x - 1) = 2$

(g)  $\log(x) - \log(x - 1) = 2$

(h)  $\log(x) - \log(x + 3) = 1$

(i)  $\ln(x^2 + 3) - \ln(x^2) = 2$

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51. Suppose I am making a pasta sauce with tomatoes and wine (and a few other things that aren't going to impact our pH much). Wine has a pH of 4 and tomatoes a pH of 5.

(a) The directions call for 1.75 cups of tomatoes and 0.25 cups of wine (who knows how good this sauce will be?!). What percent of the mixture (if I only consider tomatoes and wine) is using the tomatoes and what percent is using the wine? (Hint: Remember to find the percent consider the total amount of the item you are considering divided by the total amount you have.)

(b) What is the percent of tomatoes in decimal form? And the percent of wine in decimal form?

(c) What is the *exact* hydrogen ion concentration of my sauce?

(d) What is the pH of my sauce? (Rounding to two decimals.)

(e) If I want my pH of my sauce to be 4.75 exactly. What percent of tomatoes and what percent of wine should I include? (Rounding to two decimals.)

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52. If sound doubles in intensity, by how many units does its decibel rating increase? Show all mathematical work.



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55. Let  $f(x) = 5x^2$ , describe how each of the following functions is obtained from  $f(x)$  by applying transformations (i.e. write the given function in terms of  $f(x)$ ). Then describe how the graph of each function is obtained from the graph of  $f(x)$ .

(a)  $g(x) = 5(x - 3)^2$

(b)  $h(x) = -5(x - 3)^2$

(c)  $j(x) = 5x^2 + 3$

(d)  $m(x) = -10(x - 3)^2 + 1$

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56. Given the following functions, identify the domain and range and the vertex of all four.

(a)  $f(x) = 2x^2$

(b)  $g(x) = 2(x - 3)^2 + 4$

(c)  $h(x) = -(x - 3)^2 - 1$

(d)  $k(x) = 2(x + 1)^2 - 4$

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57. Answer the following questions:

(a) Find the vertex of  $f(x) = -2x^2 + 12x - 13$  (showing all work!).

(b) Rewrite the quadratic function  $f(x) = 3(x + 7)^2 - 9$  in standard form (showing all work!).

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58. Solve the following, showing all algebraic work:

(a)  $x^2 + 3x + 10 = 0$

(b)  $-x^2 - 3x + 60 = 0$

(c)  $x^2 - 90 = 0$

(d)  $x^2 + 16x + 64 = 0$

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59. A stage is a parabolic shape and is 10 feet in diameter and 5 feet deep.

(a) Write the equation for the shape of this stage.

(b) Where is the focal point?

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60. There is a river that cuts through land in a parabolic shape. It is found that if a man cuts his property horizontally, the east edge of his property is cut by the river a distance of 60 feet from the west edge of his property. It is also found, in the middle of this horizontal line the south edge of his property is cut by the river at a distance of 40 feet.

(a) Draw a picture of this description.

(b) Write the equation of the boundary of the river.

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61. Write the following in vertex form:

(a)  $x^2 + 6x + 81$

(b)  $3x^2 + 6x + 74$

(c)  $5x^2 + x + 15$