5. You and a friend plan to purchase cars in October. The initial value of your car will be \$34,000 and will depreciate 17% each year. The initial value of your friend's car will be \$16,500 and will depreciate 12% each year. You agree to exchange cars when their values are equal.

A. How long do you need to wait? (Give an exact value and an approximate value to the nearest month.) What is the value of your car?

Friend's car

$V_1(t) = 16500(1-0.12)^t$

$V_1(t) = 16500(0.88)^t$

Your car

$V_2(t) = 34000(1-0.17)^t$

$V_2(t) = 34000(0.83)^t$

$16500(1-0.12)^t = 34000(0.83)^t$

$\frac{16500}{34000} = \left(\frac{0.83}{0.88}\right)^t$

$t = \frac{\log 16500 - \log 34000}{\log(0.83) - \log(0.88)}$

$t \approx 12.34$

You need to wait 12 years 4 months when the value will be about \$3400.

B. What would your depreciation rate have to be in order for the values of the cars to match at the end of 7 years? Assume your friend's car still depreciates 12% each year.

$16500(0.88)^t = 34000(1-r)^t$

$16500(0.88)^7 = 34000(1-r)^t$

$(1-r)^7 = \frac{16500(0.88)^7}{34000}$

$(1-r) = \sqrt[7]{\frac{16500(0.88)^7}{34000}}$

$r \approx 0.206355$

my car would have to depreciate about 20.6% per year for the values to match at the end of seven years.

6. Prove that $y = \log_a(x)$ is proportional to $y = \log(x)$. What is the proportionality constant?

$y = \log_a x$

so $a^y = x$

$\log a^y = \log x$

$y \log a = \log x$

$y = \frac{1}{\log a} \log x$

$\frac{1}{\log a}$ is a constant

therefore $\log_a x$ is proportional to $\log x$ and the constant of proportionality is $\frac{1}{\log a}$