Analysis of a water drop on a hydrophobic surface

Mitch Wilson
Jordan Allen-Flowers
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Abstract

This paper examines the phenomenon of water dropping on a hydrophobic surface. Different behaviors are examined as a function of adjusted parameters. Data is collected and compared to previously published data, focusing on the horizontal deformation of the water drop as well as its initial contact time with the surface. The analysis shows specific behaviors that match the theoretical calculations of some authors. This data can be elaborated to future areas of similar research.

keywords: WATER BOUNCE, HYDROPHOBIC SURFACES, WEBER ANALYSIS.

1 Introduction

Our aim is to investigate the behavior of a droplet of water landing on a super-hydrophobic surface. Such surfaces are defined as those on which a stationary drop forms an angle greater than ninety degrees with the surface, see figure 1. Under the right initial conditions, a droplet of water can bounce off of the surface, like a tennis ball bouncing off the ground, as shown in figure 2. The coefficient of restitution can be quite high, reportedly of the order of 0.9 (Clanet *et al.,* 2004). Under a different set of parameters, the drop can undergo extreme deformation before bouncing, exhibiting a "crown-point" instability as shown in figure 3. A third regime, splashing, has also

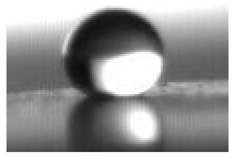

Figure 1: A drop with diameter 2.6mm rests on a super-hydrophobic surface. Note the high contact angle, which was measured to be 147 degrees.

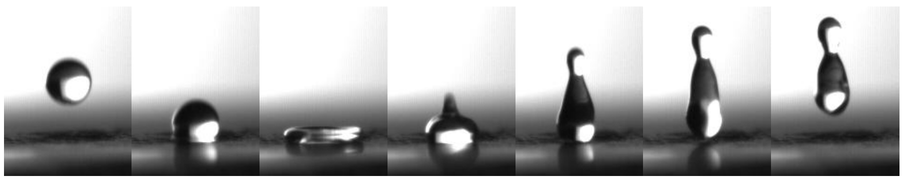

Figure 2: A bouncing drop. The drop diameter is 2.6mm and the impact velocity is .984m/s, which yields a Weber number of 17.9. The time interval between frames is 7ms. Notice the shape of the maximal deformation in the third frame.

been observed, where the droplet breaks up into many smaller satellite drops upon impact, also shown in figure 3.

Some engineering applications of this process can be found in ink-jet printing, erosion in steam turbines, nucleate boiling, fluid transport, and microfluidics (Rein 1993, Okumura *et al.,* 2003). There are also applications in non-engineering fields: soil erosion from splashing rain, spreading pesticides, liquefied meteors that hit the moon, water removal on the leaves of some plants, and blood spatters at a crime scene, just to name a few (Rein 1993, Clanet *et al.* 2004).

Although there are many factors that determine the behavior of an impacting droplet, we choose to focus our investigation on two of them: the impact velocity U and the drop diameter D, both of which are relatively easy to manipulate continuously and observe. These parameters also enable us to calculate the *Weber* number:

$$We = \frac{\rho U^2 D}{\sigma},$$

where σ and ρ are the surface tension and density of the liquid, respectively. Droplets with Weber numbers that range from ~ 1 to ~ 50 were experimen-

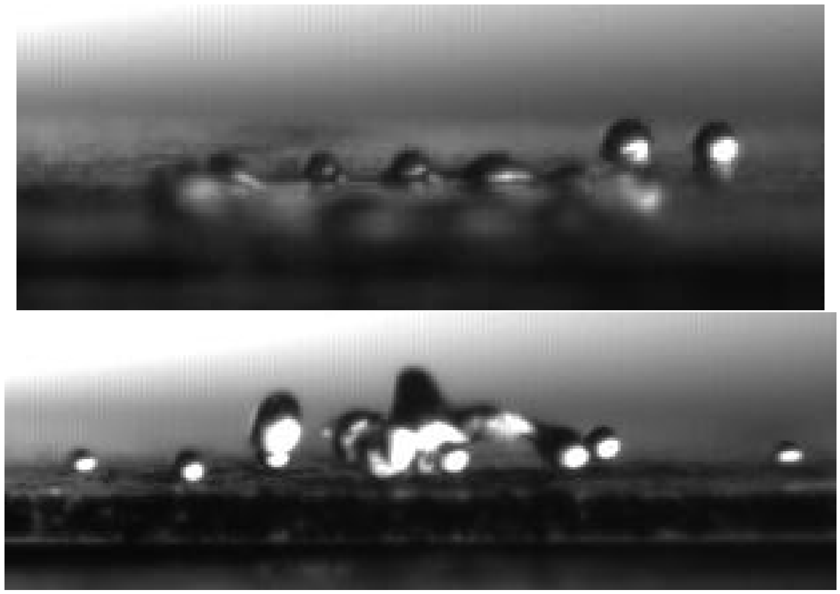

Figure 3: *Top* An example of a “crown-point” instability. The initial drop had a diameter of 2.7mm and impact velocity of 1.548m/s, which gives We=45.2. The maximum spread (seen here) is 8mm; *Bottom* An example of splashing. The Weber number is approximately 58.

tally obtained. The Weber number turns out to play an important role in determining the dynamics of the drop, because it is a ratio of the kinetic energy of the drop to its surface energy. Another parameter that factors in is the viscosity, but this (along with ρ and σ) is much harder to vary in the lab.

The measurements focus on two quantities: the maximum horizontal spread of the drop immediately after impact D_{max}, and the time that the drop is in contact with the surface, denoted by τ. In the experiments, it is verified that the scaling law put forth by Clanet *et al* (2004), which states that

$$D_{max} \sim D(We)^{\frac{1}{4}}$$

is accurate. The claim by Richard *et al* (2002) that τ is independent of U, and scales as $D^{3/2}$, is also experimentally verified.

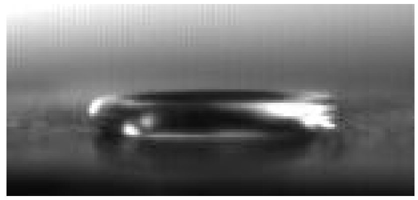

Figure 4: The shape of the drop at maximum spread resembles a gravity puddle because of its deceleration. The original diameter was 2.77mm and impact velocity .947m/s, giving We=17.2. The maximum deformation (seen here) is 5.123mm

2 Theory

Maximum horizontal spread There are different theories, many in the form of scaling laws, that describe the behavior of D_{max} in terms of the Weber number and/or the Reynolds number Re, where $Re = \rho DU/\eta$, η is the dynamic viscosity.

Chandra and Avedisian (1991) used a classical energy balance to predict D_{max}. They started by assuming that the initial kinetic energy of the drop is transferred to surface energy in the time that it takes for the drop to reach its maximum spread. Using simple physics, they balance the initial energy and the final energy, and calculate the work required for the transition between states. Doing so, they obtained

$$D_{max} \sim D(We)^{1/2}.$$

Since $We \sim U^2$, this relationship is equivalent to saying that $D_{max} \sim U$ (Okumura *et al.* 2003).

Another approach is to assume that the kinetic energy is dissipated by viscosity (Clanet *et al.* 2004). The kinetic energy of the drop is $\rho D^3 U^2$, and the dissipation scales as $\eta(U/h)D_{max}^3$, where h is the thickness of the drop at maximal spread. Combining this with volume conservation ($hD_{max}^2 \sim D^3$), one can show that $D_{max} \sim D(Re)^{1/5}$, which implies $D_{max} \sim U^{1/5}$.

Another scaling law was obtained by Clanet *et al.*, who noted that the shape of the drop at maximum deformation resembles that of a "gravity puddle," a shape that occurs when gravity overcomes surface tension, as shown in figure 4. For a stationary drop, this can only occur if the drop is larger than the capillary length $\kappa = \sqrt{\sigma/(\rho g)}$. However, as the drop impacts the surface, it undergoes an acceleration a, changing its velocity from U to 0 in some crashing time t^*, assumed to be on the order of D/U. It can be shown that the acceleration scales as $a \sim U^2/D$. Using this acceleration in place of g to calculate κ, and assuming that $h \sim \kappa$, Clanet *et al* predicted $D_{max} \sim D(We)^{1/4}$.

3 Methods

Experimental setup In order to test these theories, a specially-built apparatus was used to control the flow of water, as well as record and store videos. The setup consists of two main pieces: the physical water-drop system, and the camera with software, as shown in figure 5.

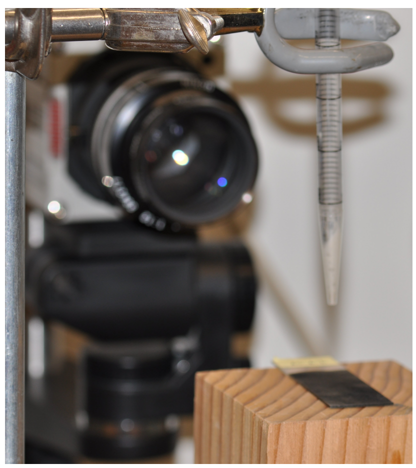

Figure 5: The water drop system positioned above a prepared slide, with the high-speed camera in the background.

The water drop experiment was performed in the Applied Mathematics Laboratory at The University of Arizona. For initial runs, a large pipette was filled with distilled water and clamped steady above the location of the test slide. The test slides were created by covering one side with a coating of candle soot and then lightly spraying with suede protectant to minimize deterioration. A number of slides were prepared on-site for testing, and others were made by lab staff. For the initial experiments, the large pipette was connected to a bulb which allowed the flow of water coming out from the pipette tip to be controlled, to the rate of one drop at a time. The slides were placed underneath the tip of the pipette, raised to a specified distance to affect the impact velocity. Upon releasing a drop of water onto the slide, any excess water was promptly removed. For later experiments where the aim was to produce a smaller drop size, a syringe needle was used instead of the pipette system. The syringe plunger was controlled manually by pressing down to release a single drop of water. This required gentle patience as the syringes prefer to expend a small stream.

For recording the data sets, a high-speed camera, Photron Motion Tools and ImageJ software, and a 1000W photographic lamp were used. The camera, provided by the lab staff, was positioned at a fixed height, facing parallel (or downward at a measurably small angle) to the contact surface. This enabled the picture of our drop to be focused and 2-dimensional as it evolved over time. The Photron Motion Tools software was used to adjust the video capture settings and to ensure a respectable video quality. Each testing day, the system was calibrated by scaling an image to ensure consistency among our data. For the experiments, the camera was set to 2000 fps to record the water-drop phenomena. For these 2000 fps videos, a large lamp was used to provide sufficient light for each frame. If necessary, a glass panel and/or diffuser film[1] was used to sufficiently diffuse the light to have a high-contrast video. The Photron Motion Tools software and the camera were able to record the moments before and after impact, which could then be cropped to include only the necessary frames.

The software program ImageJ was used to analyze the data. By using the videos, it was possible to extract a number of parameters and their values. The impact velocity was calculated by comparing the location of the drop in its frames before impact. The diameter of the drop of water was measured by taking the ratio of four times the area to the perimeter, as for a circle the result would be $(4\pi D^2/4)/(\pi D) = D$. To measure the contact time, the number of individual frames that the drop of water was in contact with the surface was counted and divided by the shutter speed.

4 Results

Figure 6 shows a plot of the normalized spread versus the Weber number on a log scale. The plot includes lines with slope 0.5 and 0.25, as well as the best fit line, which has slope 0.226. Figure 7 shows a plot of the spread

[1] Anchor Optics, model AX27428, provided by the lab staff

versus the impact velocity, along with the predictions from the three scaling laws. The slope of the best fit line is 0.49.

The data in figures 6 and 7, specifically the slope of the best fit lines, support the fact that $D_{max} \sim (We)^{1/4}$. However, if the two outliers with small U are removed from figure 7, the slope of the best fit line jumps to 0.91, suggesting that $D_{max} \sim (We)^{1/2}$. This suggests two things: that viscosity is playing a very small role in the energy dissipation (since there is no evidence to support a $U^{1/5}$ dependence); and that $D_{max} \sim (We)^{1/4}$ only holds for comparatively large Weber numbers. Clanet *et al.* also mentions that in the limit of small Weber numbers (less than one), D_{max} can be described by the viscous dissipation model.

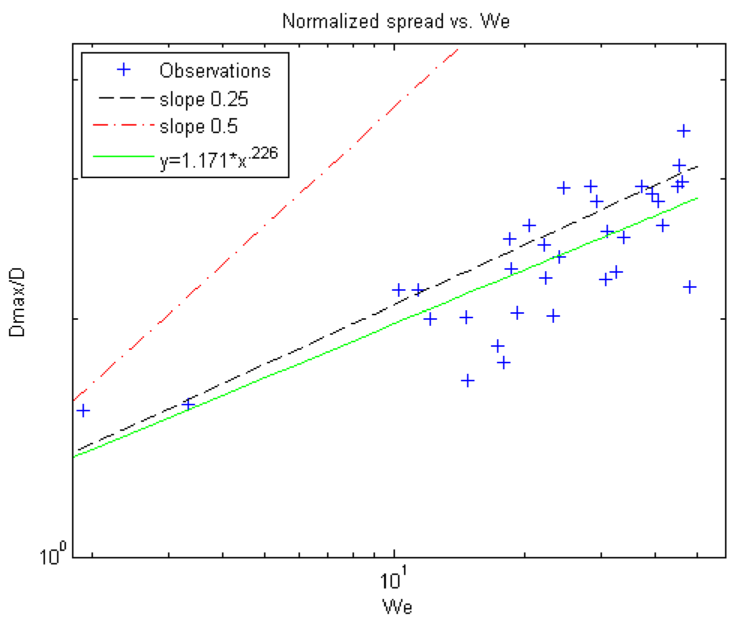

Figure 6: Testing the dependence of D_{max}/D on We on a log-log scale.

Contact time In addition to the deformation, the contact time was also investigated. This was for data which did not splash upon impact, but had some notion of bounce associated with it. This initial contact time has been examined by Richard, *et al.*, and may have implications regarding initial heat transfer and fluid flow processes.

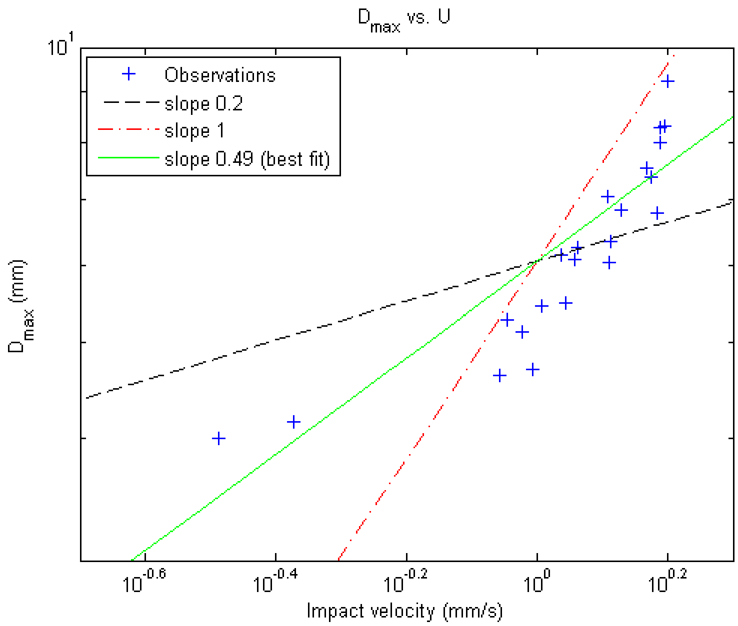

Figure 7: The three predictions for how D_{max} depends on U, shown on a log-log scale. Our data is well- described by the power law given by Clanet *et al.*

Firstly, it was investigated if there existed a correlation between the velocity of the drop and the contact time. The presumption was that a drop with higher velocity would have a shorter contact time, as suggested by the fact that the coefficient of restitution was nearly constant, so a higher velocity would result in a larger recoil velocity. In reality, this was not the result. As can be seen in figure 8, there was little correlation between the velocity and the contact time.

While the graph suggests that the contact time is directly independent from the velocity, there are other phenomena occurring at the same time. As it has been observed, the velocity provides a great influence on the Weber number, and the Weber number is related to the maximum deformation of the water drop. This deformation process requires additional time for greater Weber numbers that is not included in the slower recoil speed for lower velocities. In this sense, the contact time evens out as a quicker recoil is countered by greater deformation. This observation is consistent with Richard *et al.*, who additionally derived a power law relating the diameter of

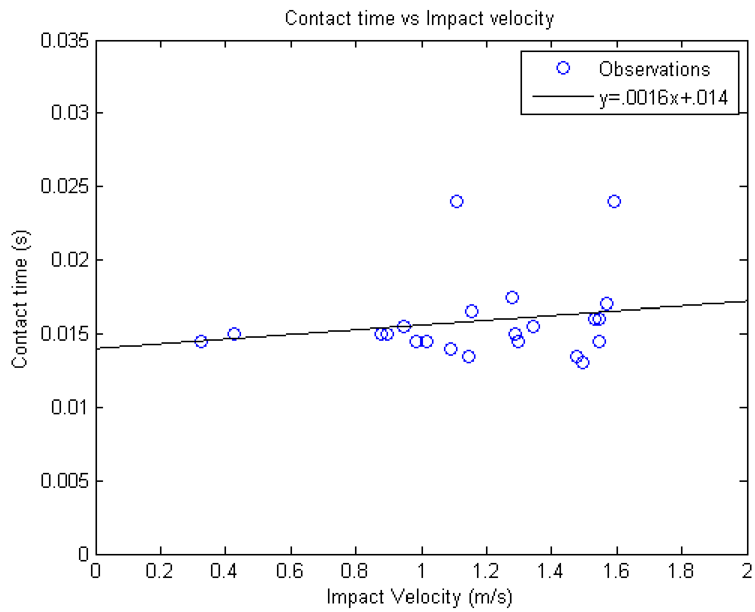

Figure 8: Contact time versus impact velocity for $D = 2.62 \pm 0.16mm$. The graph shows little correlation between τ and U. As shown in figure 9, the drop diameter plays a much bigger role in determining contact time.

the drop to its contact time, given by the following ratio:

$$\text{contact time} \propto \text{diameter}^{\frac{3}{2}}$$

As seen in figure 9, upon taking a power regression to the data, it was determined that the least-squares fit had an exponent of 1.49, which is well in agreement to that theoretically determined by Richard, *et al.*, this despite a limited range of diameter values.

It is worth noting that there is some uncertainty in obtaining our measurements. The diamater values were sensitive to the calibration for the respective day's measurements, as well as the use of ImageJ to accurately depict the perimeter and surface area. The contact time has an absolute error of $5 \cdot 10^{-4}$ seconds, equal to one frame of measurement using 2000 fps. The velocity measurements were sensitive to the shutter speed, as well, since we used a forward difference formula, along with measurements of the center of mass for each drop using ImageJ.

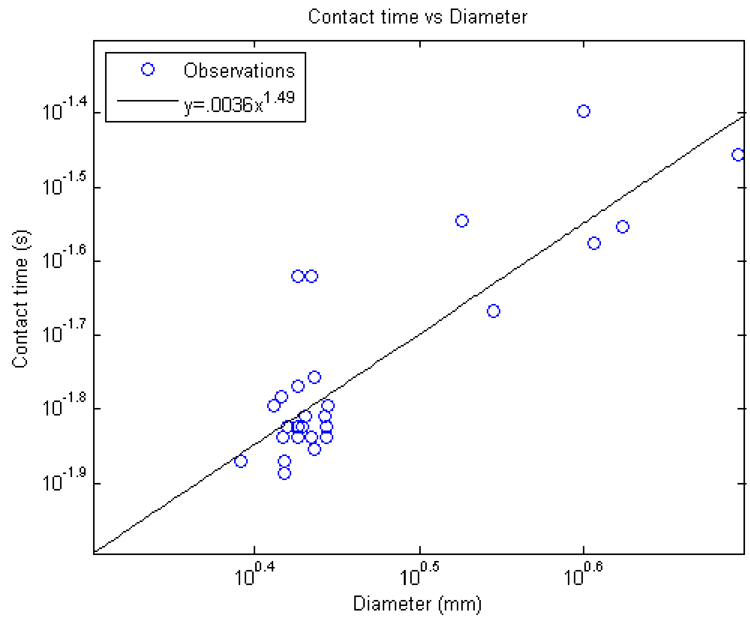

Figure 9: Our experimental observations are well-described by $\tau \sim D^{3/2}$

5 Conclusions

This paper investigated the maximal deformation of a water droplet impacting a super-hydrophobic surface, and how parameters such as impact velocity and and drop size affect the shape. Different theoretical results for the behavior of D_{max} were explored. A number of different impact scenarios were investigated, and it was found that the data is in relatively good agreement with the prediction made by Clanet *et al.*, which states $D_{max} \sim (We)^{1/4}$, at least for larger Weber numbers.

In addition, the contact time τ was examined for a number of impact scenarios. It was found that there is no correlation between τ and impact velocity U, which is the theoretical prediction of Richard *et al.*. In terms of the drop size, it was determined that τ is well-described by the power law $\tau \sim D^{3/2}$.

Future work There are a number of directions where future research can be investigated. It would be worth determining how the hydrophobic behaviors are a reflection of the surface itself. This can be measured by using a number of different hydrophobic surfaces and determining relative Weber transition numbers. Additionally, it could be useful look into the use of other liquids to see if they have any effect on the process. Discussions with lab staff encouraged the use of liquids that were less pH-balanced, like acetic acid or soapy water. It would be worthwhile to see if the viscosities of these different fluids are dismissible like that of water. Lastly, during the experimentation, a number of trends associated with the pinch-off process were observed, which was under investigation by another team. It may be beneficial to look into these phenomena to see how they affect measurable parameters related to the water-drop process.

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