HW1

Durrett 1.1 $X_n = Y_n + Y_{n-1}$, where $\{Y_i\}_{i\geq 0}$ iid fair coin tosses

Then $\{X_n\}$ is NOT a MC:

$$P(X_4 = 2 \mid X_3 = 1) = \frac{P(X_4 = 2, X_3 = 1)}{P(X_3 = 1)} = \frac{P(Y_4 = 1, Y_3 = 1, Y_2 = 0)}{P(Y_3 = 1, Y_2 = 0) + P(Y_3 = 0, Y_2 = 1)}$$

$$= \frac{1}{8} \Big/ 2 \cdot {}^1\!/_4 = \frac{1}{4}.$$

But $P(X_4 = 2 \mid X_3 = 1, X_2 = 2)$

$$= \frac{P(\{Y_4 = 1, Y_3 = 1, Y_2 = 1, Y_1 = 1\} \cap \{Y_4 = 1, Y_3 = 1, Y_2 = 0\})}{P(Y_3 = 0, Y_2 = 1, Y_1 = 1)}$$

$$= {}^0\!/_{1/8} = 0. \neq \frac{1}{4} = P(X_4 = 2 \mid X_3 = 1).$$

Intuitively, knowledge of full past in assessing later transition is not the same as knowledge of recent past in late decisions.

Durrett 1.2

5 balls	5 balls
Left	right

X_n = # white balls in left urn.

$P(j+1 \mid j) = P($choose black in left, white in right$)$

$$= \frac{n-j}{5} \cdot \frac{n-j}{5}$$

$P(j-1 \mid j) = P($choose white in left, black in right$)$.

$$= \frac{j}{5} \cdot \frac{j}{5}$$

$$P(j \mid j) = \frac{j}{5} \cdot \frac{n-j}{5} + \frac{n-j}{5} \cdot \frac{j}{5}.$$

$= P($choose same color both urns$)$

Durrett 1.6

(a)
$$\begin{array}{c c} & \begin{array}{ccc} A & B & C \end{array} \\ \begin{array}{c} A \\ B \\ C \end{array} & \begin{bmatrix} 0 & 1/2 & 1/2 \\ 3/4 & 0 & 1/4 \\ 3/4 & 1/4 & 0 \end{bmatrix} \end{array}$$

(b). $p^{(2)}(A, A)$, $p^{(2)}(A, B)$, $p^{(2)}(A, C)$

$p^{(3)}(A, B)$. Solve by finding P^2 and P^3.