(HW2)

Durrett 1.8 (d): $\{1,4\}$, $\{2,5\}$ are closed irreducible recurrent states. 3, 6 are transient since they lead to $\{1,4\}$, $\{2,5\}$ but without chance of return.

$$P_1\{T_1 > n\} = (.2)(.9)^{n-1} \to 0$$

$$\Rightarrow \rho_{11} = \underline{1}. \Rightarrow 1 \text{ recurrent.}$$

The others are similar.

(a)

$$P_x(T_y = n+1) = \sum_z P_x(X_1 = z, T_y = n+1), \quad n \geq 1$$

$$= \sum_{z \neq y} P_x(X_1 = z, T_y = n+1)$$

since $T_y \geq 2$

$$= \sum_{z \neq y} P_x(X_1 = z, X_2, \dots, X_n \neq y, X_{n+1} = y)$$

$$= \sum_{z \neq y} p(x,z) P_z(X_1, \dots, X_{n-1} \neq y, X_n = y)$$

$$= \sum_{z \neq y} p(x,z) P_z(T_y = n)$$

(b) and (c) Follow from (a) by noting $P_x(T_y \leq n+1) = \sum_{k=1}^{n+1} P_x(T_y = k)$ etc.

$$\rho_{13} = p(1,3) + \sum_{j=2,1,4} p(1,j)\rho_{j3}$$

$$= p(1,3) + p(1,1)\rho_{13} + p(1,2)\rho_{23} + p(1,4)\rho_{43}$$

$\to 0$ as 4 absorbing

$$\rho_{23} = p(2,3) + p(2,1)\rho_{13} + p(2,2)\rho_{23}$$

Solve!