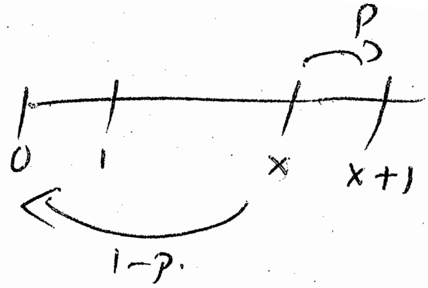
HW 4

(1)



$\pi(0) = (1-p)\pi(0) + (1-p)\pi(1) + \dots$

$= (1-p)\sum_{x=0}^{\infty}\pi(x) = 1-p$ if π exists.

$\pi(1) = p\,\pi(0)$

$\pi(2) = p\,\pi(1)$ $\Rightarrow$ $\pi(x) = p^x\pi(0)$ $x \geq 0$.

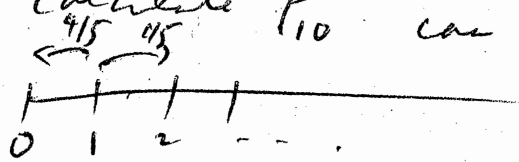
$\vdots$ $\Rightarrow$ $\pi(x) = p^x(1-p).$ $x \geq 0$

check that π is a probability: $\sum_{x=0}^{\infty}\pi(x) = \sum_{x=0}^{\infty}p^x(1-p)$

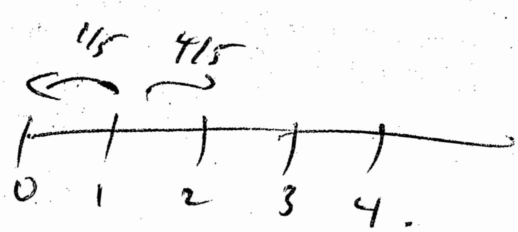
$= \frac{1-p}{1-p} = 1$ ✓

(2) $\rho_{00} = \frac{1}{5}\rho_{10} + \frac{4}{5}\rho_{-10}$

To calculate ρ_{10} can use B-D chain



To calculate ρ_{-10} use (after mapping x to $-x$).



Then $\rho_{00} = \frac{1}{5}\cdot 1 + \frac{4}{5}\cdot\frac{1}{4} = \frac{2}{5}.$

(3) states 2 = two working.

1 = one working

0 = none working.

$$\begin{array}{c|ccc} & 0 & 1 & 2 \\ \hline 0 & 0 & 0 & 1 \\ 1 & .05 & .95 & 0 \\ 2 & 0 & .02 & .98 \end{array}$$

(1) long run fraction one bulb working

$$= \pi(1) = \frac{100}{5} \cdot \frac{1}{71}$$

(2) expected time between bulb replacements

$$E_0 T_0 = \frac{1}{\pi(0)} = 71.$$

$$\pi(0) = (.05)\pi(1)$$
$$\pi(1) = (.02)\pi(2) + (.95)\pi(1)$$
$$\pi(2) = 1 \cdot \pi(0) + (.98)\pi(2).$$

$$\Rightarrow \quad \pi(2) = \frac{100}{2}\pi(0).$$
$$\pi(1) = \frac{100}{5}\pi(0).$$

$$\Rightarrow (1 + 50 + 20)\pi(0) = 1$$
$$\Rightarrow \pi(0) = \frac{1}{71}.$$

$$\Rightarrow \pi(1) = \frac{100}{5} \cdot \frac{1}{71}$$
$$\pi(2) = \frac{100}{2} \cdot \frac{1}{71}$$

(4) Compute waiting times for HHH, HHT, HTT, HTH.

(1) Write down the 8×8 transition matrix. Compute stat dist is $\langle \frac{1}{8}, \cdots, \frac{1}{8} \rangle$.

(2). The $E_\square T_\square = \frac{1}{\pi_\square} = 8.$ all choices of $\square$.

(3) So, $8 = E_{HHH} T_{HHH} = \underset{\substack{\uparrow \\ \text{get a H}}}{\frac{1}{2}} \cdot 1 + \underset{\substack{\uparrow \\ \text{get a T.}}}{\frac{1}{2}} (1 + E T_{HHH})$. $\Rightarrow \boxed{E T_{HHH} = 14.}$

Also, $\boxed{E_{HHT} T_{HHT} = 8}$ since starting from HHT is the same as starting with no info.

$\boxed{E_{HTT} T_{HTT} = 8}$ similarly

$\Rightarrow \boxed{E_{HTH} = 8.}$

$$8 = E_{HTH} T_{HTH} = \underset{\substack{\uparrow \\ TH}}{\frac{1}{4}}(2) + \underset{\substack{\uparrow \\ TT}}{\frac{1}{4}}(2 + E T_{HTH}) + \underset{\substack{\uparrow \\ HT}}{\frac{1}{4}}(2 + E T_{HTH}) + \underset{\substack{\uparrow \\ HH}}{\frac{1}{4}}(2 + \underset{\substack{\uparrow \\ \text{Same as starting with HHH}}}{\overset{\substack{8 \\ \parallel}}{E_{HTH} T_{HTH}}})$$