Solns HW 6

2.30 Prob (one customer)

(a) $= \lambda s e^{-\lambda s}$

(b) $\lambda e^{-\lambda s} = \lambda^2 s e^{-\lambda s}$.

$\Rightarrow 1 = \lambda s. \Rightarrow s = \frac{1}{\lambda}.$

$\Rightarrow \lambda s e^{-\lambda s} = e^{-1}$

4.2

$$A = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{bmatrix} -1 & 0 & 1 & 0 \\ 2 & -3 & 0 & 1 \\ 0 & 2 & -2 & 0 \\ 0 & 0 & 2 & -2 \end{bmatrix} \end{array}$$

$\bar{a} = \left(\frac{4}{12}, \frac{2}{12}, \frac{3}{12}, \frac{3}{12}\right)$

rate of sales $= \frac{8}{12} \cdot \frac{1}{1/\lambda} = \frac{8}{12} \cdot 2 = \frac{16}{12}$

2.46 (a) mean $= 100 \cdot \frac{\lambda}{3} + 400 \cdot \frac{2\lambda}{3}$, $\lambda = 6$

(a) st dev $= \sqrt{\frac{\lambda}{3} \cdot 100^2 + \frac{2\lambda}{3} \cdot 400}$.

(b) $P(5 \text{ speeding} \mid \text{DWI}) = \frac{\left(\frac{2\lambda}{3}\right)^5 e^{-2\lambda/3}}{5!} \cdot \frac{\frac{\lambda}{3} e^{-\lambda/3}}{1!}$

(c) $P(A) = e^{-\lambda/2}$, $P(A \mid N=5) = \left(\frac{1}{2}\right)^5$.

$= e^{-3}$

$P(A)$ greater

2.51 $(X_0, X_1, X_2, X_3) \sim$ indep

Poisson with parameters $(300 \cdot \lambda(1-P_1)(1-P_2)$,

$300 \cdot \lambda P_1 (1-P_2)$,

$300 \cdot \lambda P_2 (1-P_1)$,

$300 \lambda P_1 P_2)$.

$80 = 300 \cdot \lambda P_1 P_2$

$0 = 300 (\lambda P_1 (1-P_2) + \lambda P_1 P_2) = 300 \lambda P_1$

$0 = 300 \lambda P_2$

$P_1 = \frac{80}{120}$, $P_2 = \frac{80}{100}$, $\lambda = \frac{1}{2}$, $X_0 \sim 10$.