Solns 7

(1) $\bar{\pi} \begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{pmatrix} -4 & 2 & 2 \\ 3 & -4 & 1 \\ 5 & 0 & -5 \end{pmatrix} \end{array} = \bar{0}$ and $\pi_A + \pi_B + \pi_C = 1.$

$\Rightarrow \pi_A = \frac{1}{2}, \quad \pi_B = 1/4, \quad \pi_C = 1/4.$

Average # trips $B \to A$ in a year

$=$ fraction of time in $B \times 1\text{yr} \times$ rate $\frac{3 \text{ trips}}{D \cdot \text{year}}$

$= \frac{3}{4D}$ trips in a year.

Here if $D = \frac{1}{12}$, then 9 trips per year.

(2)

$$\begin{array}{c|c} & \begin{array}{cccc} 0 & 1 & 2 & 12 \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 2 \\ 12 \end{array} & \begin{bmatrix} -2 & 2 & 0 & 0 \\ 0 & -4 & 4 & 0 \\ 2 & 0 & -4 & 2 \\ 0 & 2 & 4 & -6 \end{bmatrix} \end{array}$$

$\bar{\pi} = \left(\frac{1}{3}, \frac{2}{9}, \frac{1}{3}, \frac{1}{9}\right)$

(a) Customer enters if server 1 if not busy. So, proportion is $\pi_0 + \pi_2 = \frac{2}{3}$.

(b) proportion who visit 2

$= \pi_0 + \pi_2 \cdot$ Prob of server 2 being empty before served.

$= \frac{1}{3} + \frac{1}{3} \cdot \frac{2}{6} = \frac{4}{9}.$

(3) $\quad 0 \quad + \quad -$

$$\begin{pmatrix} -3 & 1 & 2 \\ +3 & -3 & 0 \\ 4 & 0 & -4 \end{pmatrix} \qquad \bar{\pi} = \left(\frac{6}{11}, \frac{2}{11}, \frac{3}{11}\right)$$

free $\frac{6}{11}$ of time

O_2 $\frac{2}{11}$ of time

CO $3/11$ of time.

(4) $-B = \begin{pmatrix} 4 & -1 \\ 0 & 5 \end{pmatrix}$

$$\vec{t} = (-B)^{-1}\vec{1} = \frac{1}{20}\begin{pmatrix} 5 & 1 \\ 0 & 4 \end{pmatrix}\vec{1} = \begin{pmatrix} \frac{6}{20} \\ \frac{4}{20} \end{pmatrix}$$

$$= \begin{pmatrix} E_B[T_A] \\ E_C[T_A] \end{pmatrix}.$$

$\Rightarrow$ Time to return after leaving A $= \frac{1}{2}\left(\frac{6}{20} + \frac{4}{20}\right) = \frac{1}{4}.$

equal prob to go to B or C

(b) $q(A)E_A T_A = \frac{1}{\pi_A} = 2$ (from #1)

$\Rightarrow$ as $q(A) = 4$, $E_A T_A = \frac{2}{4} = \frac{1}{2}$

Now, wait at A for $\frac{1}{4}$ units. $\Rightarrow$ time to return $= \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$