HW 2 Math 523a Fall 2016

Due Sep. 30, 2016

1. (Folland 6). Complete the proof of Thm 1.9.

2. (Folland 19) Let μ^* be an outer measure on Ω induced from finite premeasure μ_0. If $E \subset \Omega$, define the inner measure of E to be $\mu_*(E) = \mu_0(\Omega) - \mu^*(E^c)$. Show E is μ^*-measurable iff $\mu_*(E) = \mu^*(E)$.

3. (Folland 23) Let $\mathcal{A}$ be the collection of the empty set and finite unions of sets of the form $(a, b] \cap \mathbb{Q}$ where $-\infty \leq a < b \leq \infty$.
(a) Show $\mathcal{A}$ is an algebra on $\Omega = \mathbb{Q}$.
(b) Show that the σ-field generated by $\mathcal{A}$ is $\mathcal{P}(\mathbb{Q})$.
(c) Define μ_0 on $\mathcal{A}$ by $\mu_0(\emptyset) = 0$ and $\mu_0(A) = \infty$ for $A \neq \emptyset$ in $\mathcal{A}$. Show μ_0 is a premeasure on $\mathcal{A}$, and that there is more than one measure on $\mathcal{P}(\mathbb{Q})$ whose restriction to $\mathcal{A}$ is μ_0.

4. (Folland 29) Let E be a Lebesque measurable set, and m denote Lebsesque measure.
(a) If $E \subset N$ where N is the nonmeasurable set described in Folland 1.1, then $m(E) = 0$.
(b) If $E \subset [0, 1]$ and $m(E) > 0$, then E contains a nonmeasurable set. (Note that $E = \cup_{r \in R} E \cap N_r$ in the notation of 1.1).

5. (a useful lemma; compare Prop 1.20) Let $(\Omega, \mathcal{F}, \mu)$ be a finite measure space. Suppose $\mathcal{F} = \sigma(\mathcal{A})$ where $\mathcal{A}$ is an algebra. Then, for $B \in \mathcal{F}$, we have for each $\epsilon > 0$ that there exists $A_\epsilon \in \mathcal{A}$ such that $\mu(B \triangle A_\epsilon) < \epsilon$.