HW 4 Math 523a Fall 2016

Due Wed., Oct. 12, 2016

1. (Folland 3). Complete carefully the argument that if $\{f_n\}$ is a sequence of measurable functions, then $\{\omega : \lim f_n(\omega) \text{ exists}\}$ is a measurable set.

2. Folland 9, in Section 2.1

3. (Folland 14) If $f \in L^+$, let $\lambda(E) = \int_E f d\mu$ for $E \in \mathcal{F}$. Then λ is a measure on $\mathcal{F}$ and for any $g \in L^+$, $\int g d\lambda = \int f g d\mu$. Hint: First suppose that g is simple.

4. Folland (17) Assume Fatou's Lemma and deduce MCT from it.

5. Folland (20) Show the Generalized DCT as in Folland.

6. Folland (21) Suppose f_n, f are integrable functions, and $f_n \to f$ pointwise. Then, $\int |f_n - f| d\mu \to 0 \Leftrightarrow \int |f_n| d\mu \to \int |f| d\mu$.