

Europäisches Forum Alpbach

18 August, 2003

Chaos in the real world

Chaos everyday

- Water dripping from a faucet.
- Convection rolls in a heated kettle.
- The global weather ?
- Populations of some species.
- The heart.
- The brain ?
- Meteorites.
- Economic systems?

Time Series

I have a stream of data from a system. Can I find out if the system is chaotic?

A sequence of data is called a **Time series**.

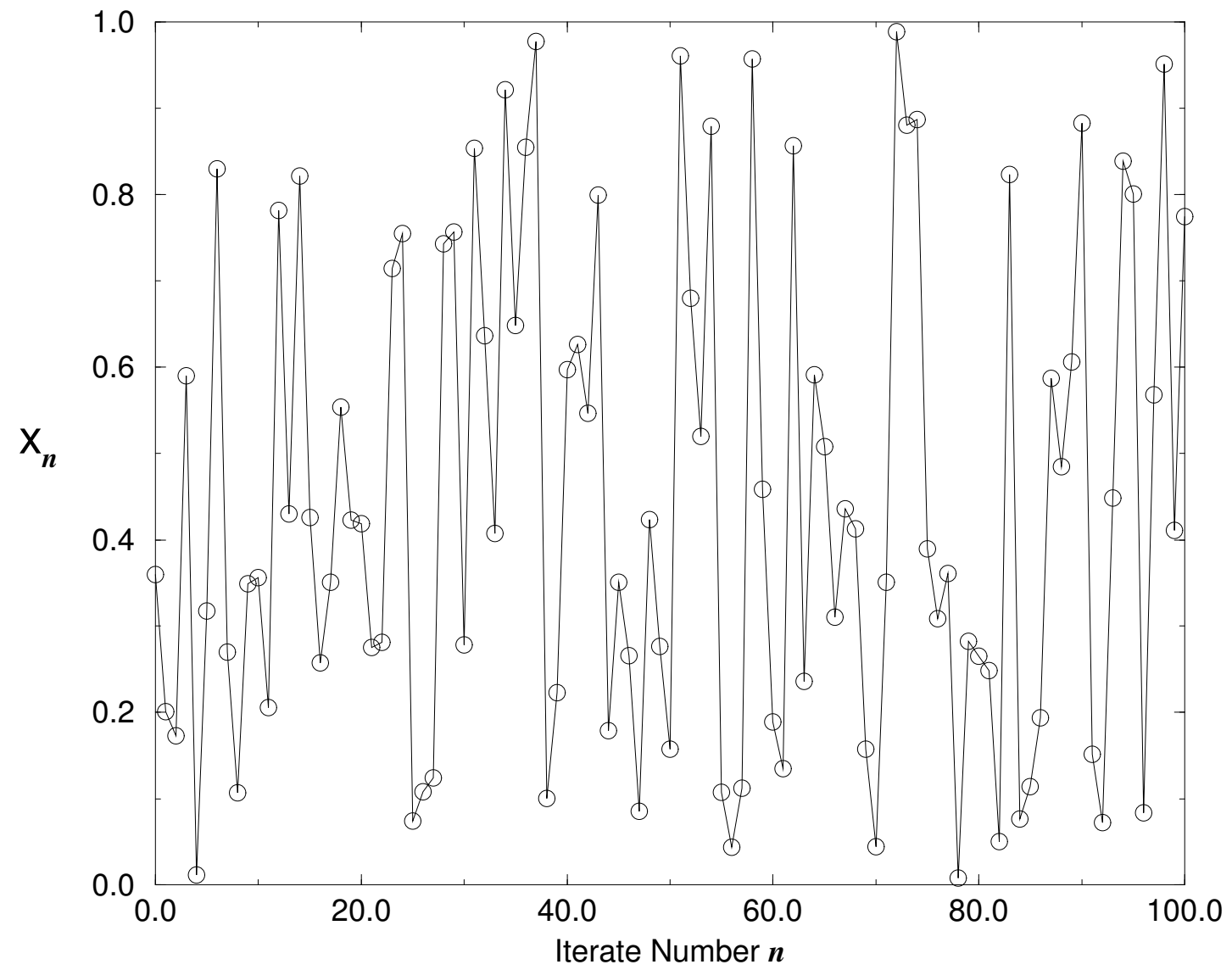
Examples :

- The temperature in Alpbach.
- An EEG signal.

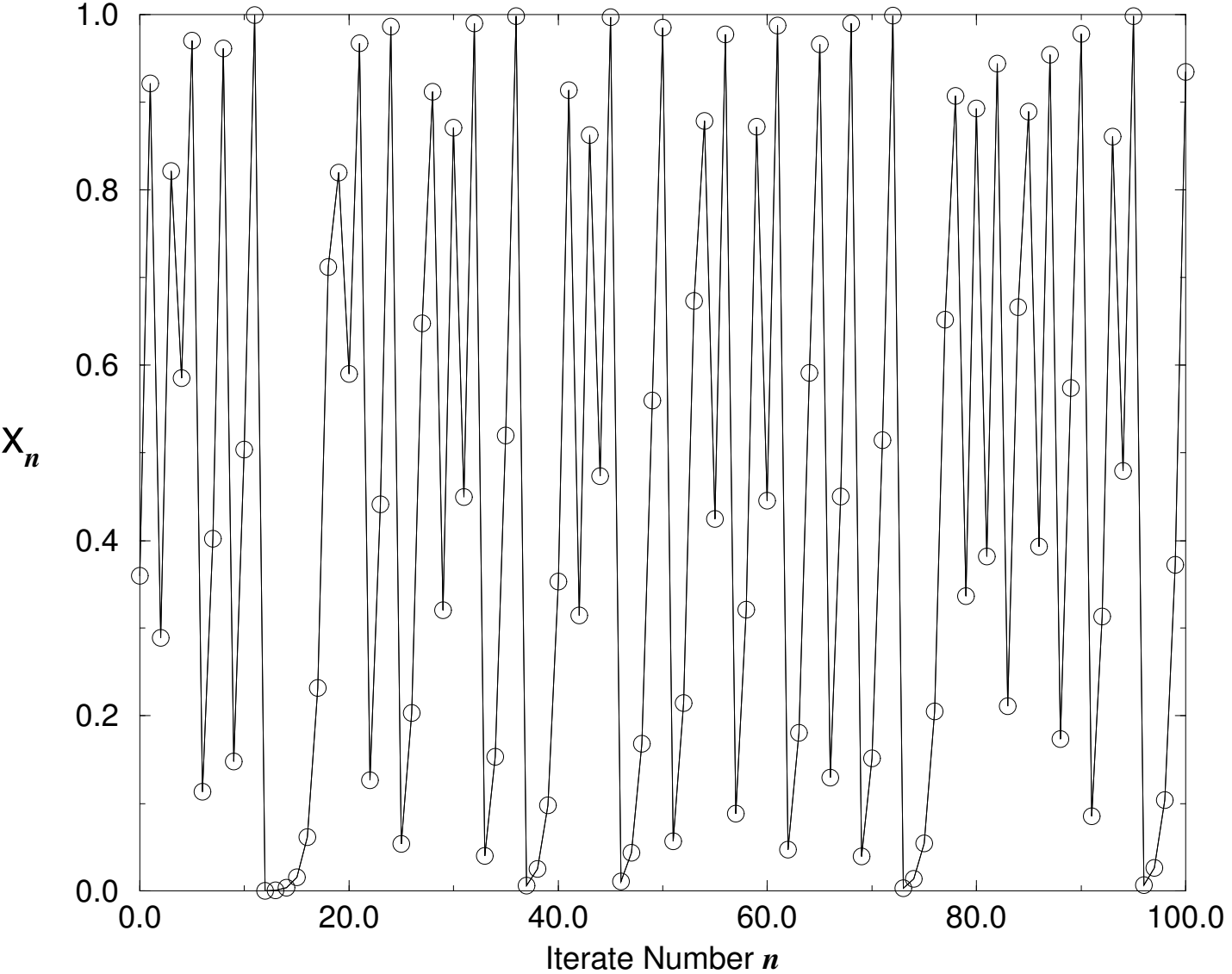
We can do one of the following :

- Attempt to find the rules governing the dynamics of the system.
- Check if the orbit is on a Strange Attractor.

Random Time series



Logistic Map : $r = 4.0$



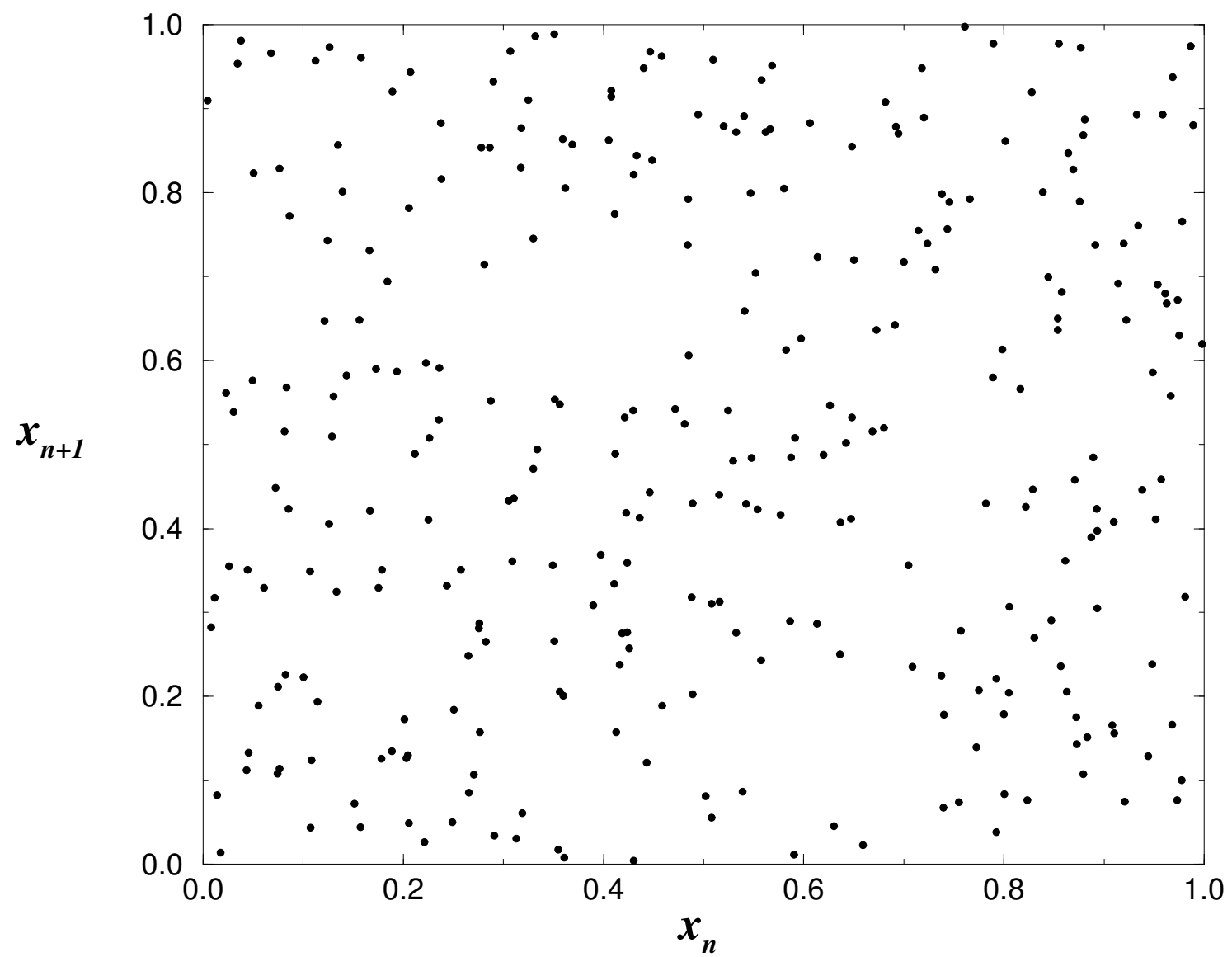
Embedding

Take the time series and plot the trajectories in a phase space. Do this by plotting the phase variables at a later time as a function of the variables at an earlier time. This is called a **return map**.

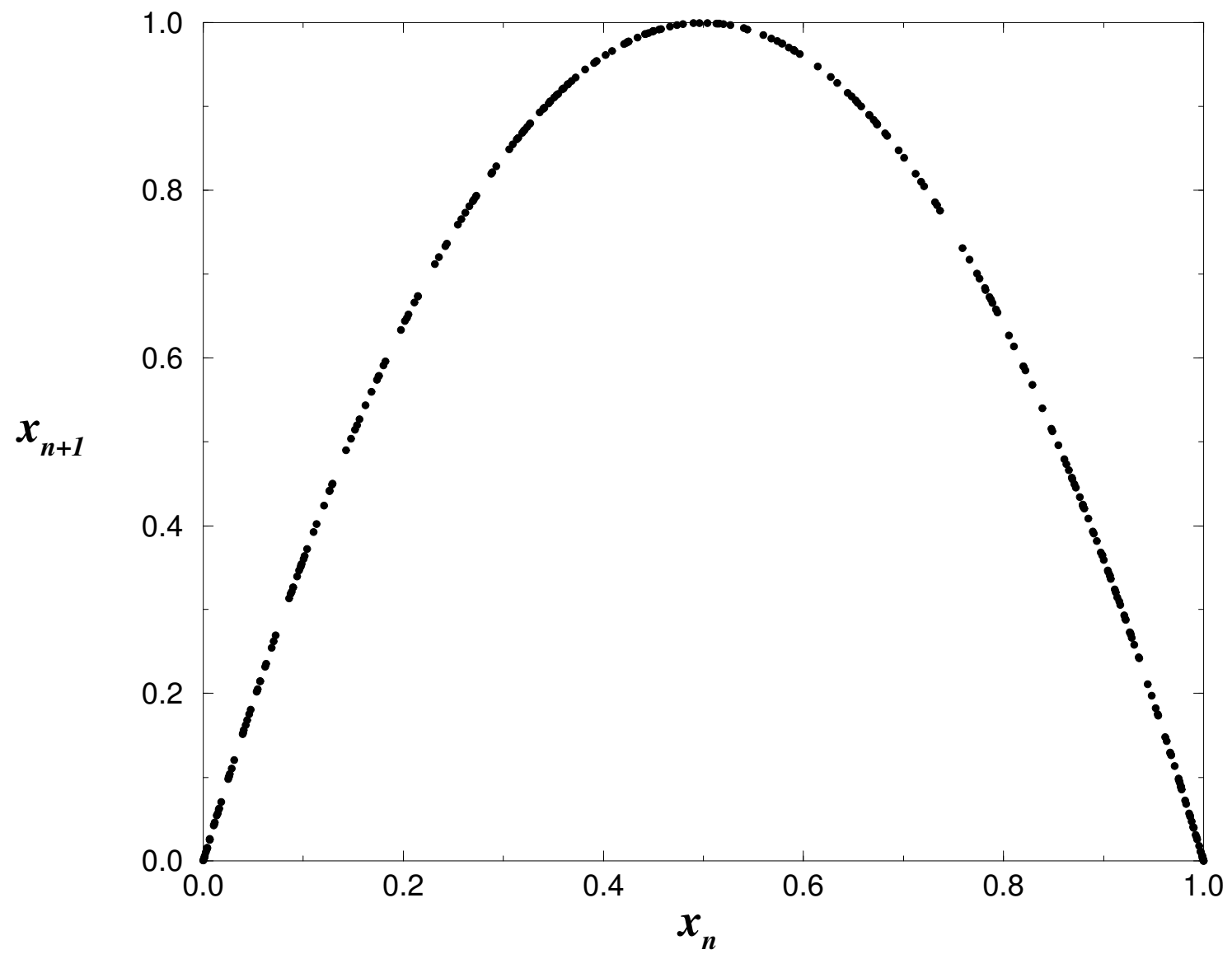
A typical trajectory will wander all over its attractor. This procedure will reveal if the system is deterministic and will reveal the dynamics on the attractor.

- If we are given the time series of an orbit of the logistic map, the data points will all fall on a smooth curve if we plot x_{n+1} as a function of x_n .
- If on the other hand we look at a random time series, the data will not fall on a single curve.
- Water dripping from a faucet.

Return Map for the random time-series



Return Map for the time-series from the logistic map



Is the weather chaotic ?

- This is a very hard question.
- There is no record of the weather ever repeating itself.
- The weather sometimes gets to a state where it is close to a state it has been in earlier, at least over a localized region. This is called an analogue.
- The weather does not have the same history starting at an analogue. this would indicate a sensitivity to initial conditions.
- It would seem that the weather is actually an infinite dimensional system because we presumably need to specify the initial conditions at every point in the earth's atmosphere.

- The atmosphere is a dissipative system and the weather has settled into an attractor.
- Current weather prediction is through the use of mathematical models on computers. We “know” the equations that weather obeys.
- The number of variables in these models, *i.e.*, the size of the phase space is enormous. Some of the good models use five million variables. It is not possible to do an embedding or to reconstruct the attractor from the weather data given the size of the phase space.
- The computer models show sensitivity to changes in initial conditions. It is very likely that they are chaotic.
- It seems reasonable that the global weather system is also chaotic.

Astrophysical Chaos

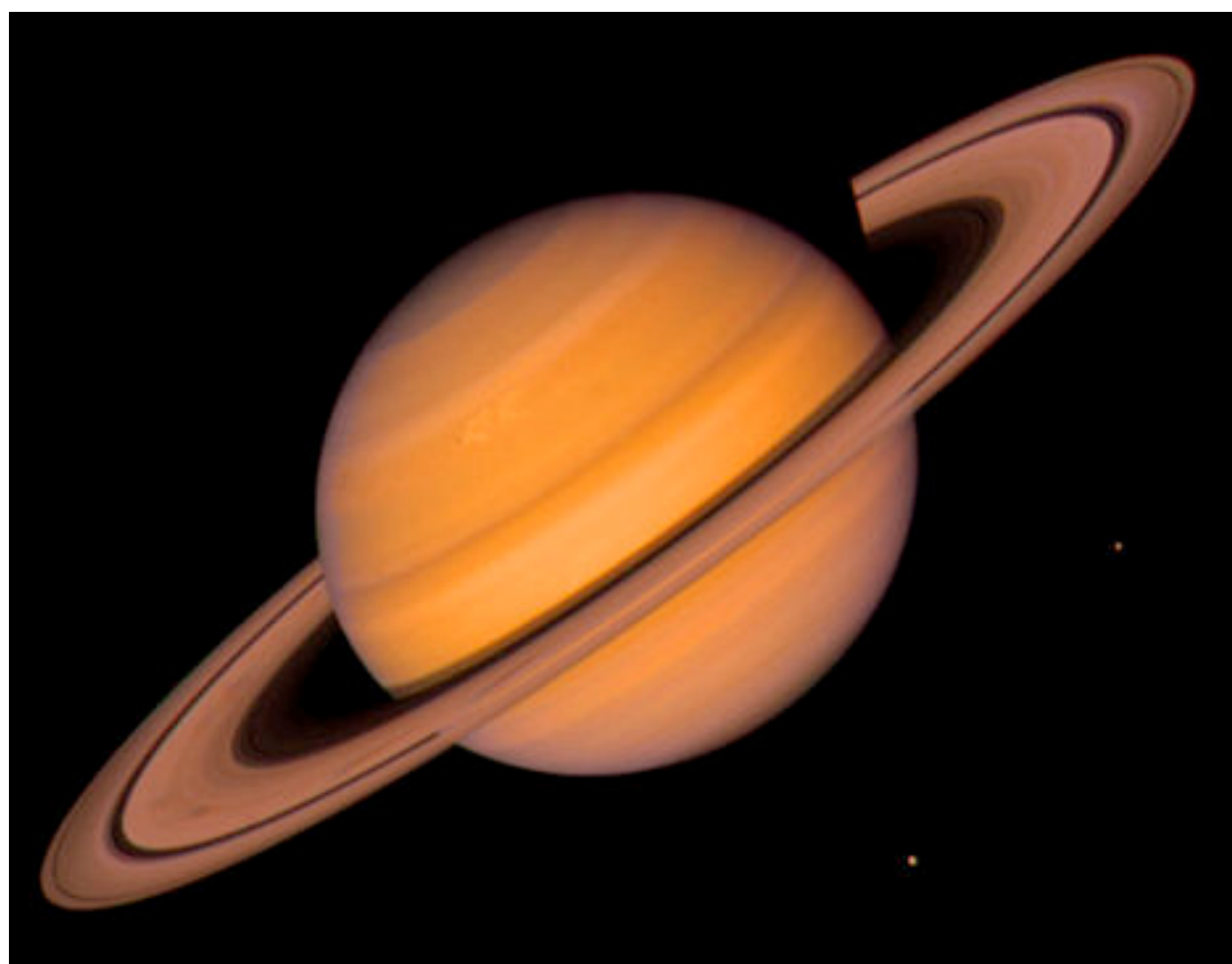
Hill's reduced problem - A light satellite revolving around two massive objects.

This problem has a long history. Henri Poincare's study of this problem laid the foundations of two branches of modern mathematics- Topology and Dynamical System Theory.

The orbit is chaotic for a variety of initial conditions.

This is not a cause for alarm because there are also stable orbits coexisting with chaotic orbits (this is a Hamiltonian system) and the earth is not going to go chaotic anytime soon.

There however are objects in the solar system that are on these chaotic trajectories and every so often they slam into the earth's atmosphere. They are meteorites.





Chaos in Biology

From Clocks to Chaos - L. Glass and M. C. Mackey.

- Population dynamics.
- Chaos in the heart - Arrhythmia.
- Chaos in the brain - Epilepsy.
- Epidemiology.

Population dynamics - The logistic map is a caricature of a rule governing population dynamics. It displays chaos.

Chaos in the Heart

The heart has its own pacemaker, the SA-Node.

Both the SA-Node and external Pacemakers work by frequency-locking.

The heart's cycle feeds back to its pacemaker keeping a regular rhythm.

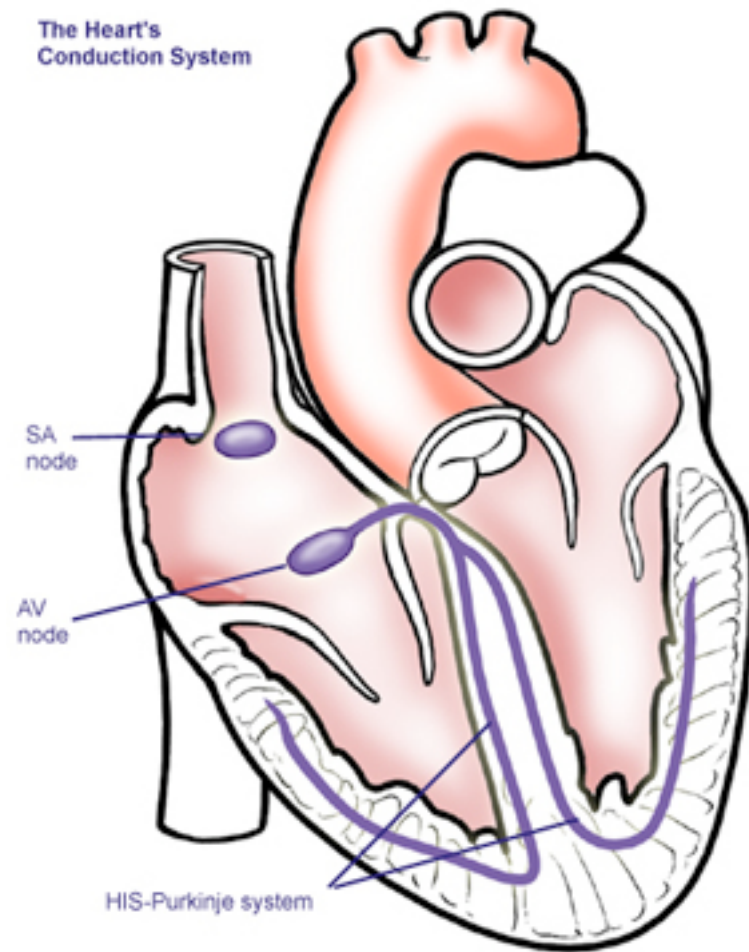
If this rhythm becomes chaotic, it leads to arrhythmia, which can lead to cardiac arrest and to death.

Frequency-locking can be seen in the circle map

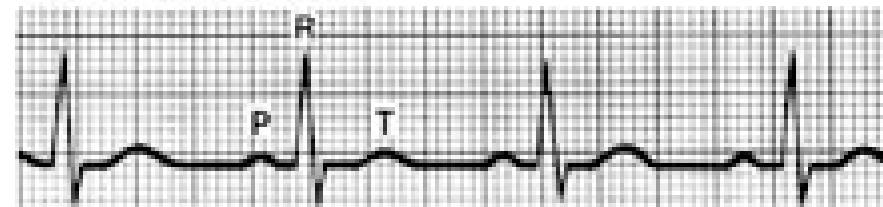
$$\theta_{n+1} = \theta_n + \omega + k \sin(\theta_n)$$

L. Glass proposed a model for the frequency locking in the heart. The theory agrees well with experiments on cells from a chicken heart.

The Heart's
Conduction System



Normal Sinus Rhythm



Chaos in the Social Sciences

- It is not clear that society or individuals behave deterministically.
- The number of variables is large so that the phase space is enormous.
- As in the case of weather, there is no low dimensional chaos. No long-range prediction.
- Unlike the case of weather, there are no known underlying equations. Even short range prediction is hard.
- Chaos has a very important philosophical role. It challenges some of the existing notions and brings up new paradigms in the description of social systems.

Chaos in Economics ?

- No Low-dimensional Chaos.
- It is not easy to predict economic systems or to beat the stock market.
- It is not currently possible to use chaos theory quantitatively in the study of economic systems.
- There are other notions that come up in the study of nonlinear complex systems that may describe social, biological and economic systems. For example, it is possible that these systems are self-organized.
- Some economic quantities obey interesting (and not very well understood) scaling laws.

Chaotic Attractors

Dissipative dynamical systems have attractors.

If these attractors are *fractal*, they are called *Strange Attractors*.

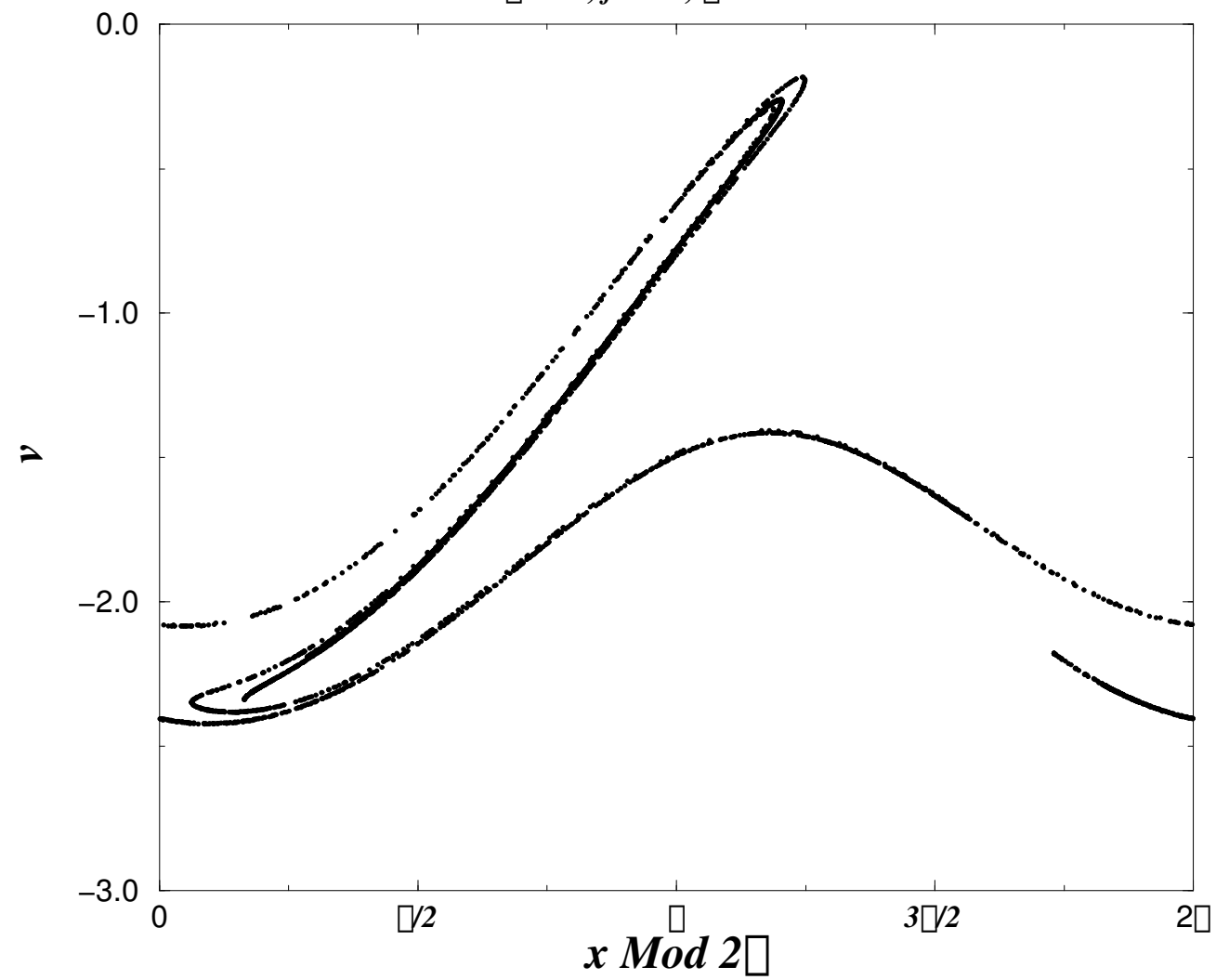
A strange attractor usually corresponds to a chaotic orbit being attracting.

Examples :

- The forced-damped pendulum.
- Periodically modulated double well.

Poincare Section for the Forced-damped pendulum

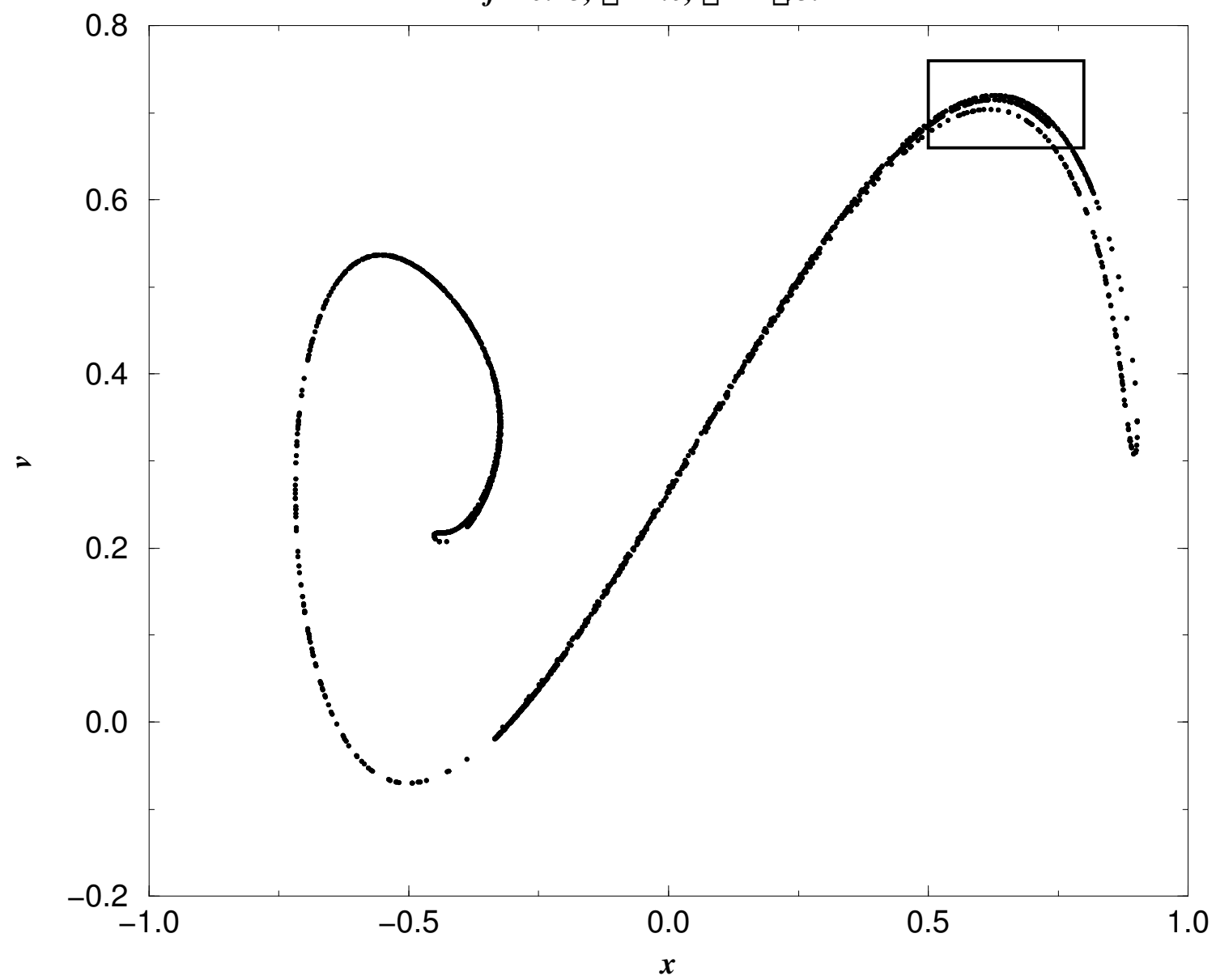
$$\gamma = 0.5, f = 1.8, \omega = 0.63.$$



Poincare section for the forced damped pendulum.

Stroboscopic section for the double well

$f = 0.75, \mu = 1.0, \gamma = 2\pi/5.$



The butterfly effect *vs.* attractors

After some time, the transient effects from the initial conditions die out and a dissipative system settles into its [asymptotic state](#).

The asymptotic state is a chaotic trajectory if the system has a strange attractor.

All chaotic trajectories are very sensitive to small changes.

Thus, we cannot predict a chaotic trajectory exactly, even if we know the attractor.

The attractor however gives us a lot of information about the dynamics.

