Math 466 - Spring 18 - Homework 7

1. Suppose that the population is uniformly distributed on $[0, \theta]$. Find the minimum variance unbiased estimator (MVUE) of θ. Hint: problem 6 from homework 3 might be useful.

2. (Book 9.59) The number of breakdowns Y per day for a certain machine has a Poisson distribution with mean λ. The daily cost of repairing those breakdowns is given by $C = 3Y^2$. If $Y_1, Y_2, \cdots, Y_n$ are the observed number of breakdowns for n independently selected days, find the MVUE for $E(C)$.

3. (This problem will count as two problems since it has so many parts.) Suppose that the population has the following pdf:

$$f(y) = \begin{cases} e^{-(y-\theta)} & \text{if } y \geq \theta \\ 0 & \text{otherwise} \end{cases}$$

Let $U_1 = \min\{Y_1, \cdots, Y_n\}$ and $U_2 = \sum_{i=1}^n Y_i$.
(a) Show that the pdf of U_1 is

$$f(y) = \begin{cases} ne^{-n(y-\theta)} & \text{if } y \geq \theta \\ 0 & \text{otherwise} \end{cases}$$

(b) Show that $U_1 - 1/n$ and U_2/n are both unbiased estimators of θ.
(c) Find the variance of each of the unbaised estimators in part (b).
(d) One of U_1, U_2 is a sufficient statistic. Which one? You should explain your answer.
(e) Find the MVUE of θ. Note that your answer should be consistent with your answer to (c).

4. (book 9.69) Suppose that the population has the following pdf:

$$f(y) = \begin{cases} (\theta + 1)y^\theta & \text{if } 0 \leq y \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

The parameter is restricted to the range $\theta > -1$.
(a) Find the method of moments estimator $\hat{\theta}$ for θ.
(b) Show your answer to (a) is a consistent estimator.
(c) Show that $U = -\sum_{i=1}^n \ln Y_i$ is a sufficient statistic.
(d) Is your $\hat{\theta}$ from part (a) a function of U?

5. Suppose that the population has the following pdf:

$$f(y) = \begin{cases} \beta^{-1} e^{-(y-\theta)/\beta} & \text{if } y \geq \theta \\ 0 & \text{otherwise} \end{cases}$$

where β and θ are unknown parameters. This is a shifted exponential distribution. If Y has the usual exponential distribution with mean β, then $Y + \theta$ has the above distribution.

(a) Find the mean and variance of the above pdf. (Your answers should depend on β and θ.)

(b) Use the method of moments to find estimators $\hat{\beta}$ and $\hat{\theta}$.

(c) Show that your estimators are consistent.