Two sets are equinumerous if they have the same number of elements. A set has five elements if it is in one-to-one correspondence with $\{1, 2, 3, 4, 5\}$.

In general, we decide if two sets have the same number of elements by trying to find a 1-1 correspondence between them.

So $\{0, 1, 2, 3, \ldots\}$ is in 1-1 correspondence with the set $\mathbb{N} = \{1, 2, 3, 4, \ldots\}$ using $f(n) = n + 1$. A set in 1-1 correspondeince with $\mathbb{N}$ is said to be denumerable.

Are the integers denumerable? Yes because we can list them:

$$0, -1, 1, -2, 2, -3, 3, \ldots$$

If we can list the elements of a set then we can map each element to its position in the list. This gives a 1-1 correspondence with $\mathbb{N}$. Here the function is

$$f : \mathbb{N} \to \mathbb{Z}, \quad n \mapsto \text{the } n\text{-th integer in the list}$$

$$f : \mathbb{N} \to \mathbb{Z}, \quad n \mapsto \begin{cases} -n/2 & n \text{ even} \\ (n-1)/2 & n \text{ odd} \end{cases}$$

$\mathbb{R}$, $\mathbb{Q}$. Are these denumerable? Can I write a list of all the real numbers? Can I write a list of all the rational numbers?

It turns out that the rationals are denumerable and the reals are not.

To list all the rationals, we write them in an array:

$$\begin{array}{ccccc} 0 & \frac{1}{1} & \frac{1}{2} & \frac{1}{3} & \ldots \\ \\ 2 & \frac{2}{2} & \frac{2}{3} & \frac{2}{4} & \ldots \\ \\ 3 & \frac{3}{2} & \frac{3}{3} & \frac{3}{4} & \ldots \end{array}$$

Then we count from top left, following the diagonals back and forth:

$$0, 1, 2, 3, \frac{1}{2}, \frac{1}{3}, \frac{2}{3}, \frac{3}{2}, 4, \ldots$$