More examples of separable differential equations

Consider the differential equation

$$v' = 9.8 - k_0 v^2.$$

Rewriting it in the form

$$v' = -k_0(v^2 - \frac{9.8}{k_0}) = k_0(v^2 - \alpha^2),$$

we see that it is similar in form to the equation

$$x' = x^2 - 1$$

whose solution we have already found to be

$$x' = \frac{1 - Ce^{2t}}{1 + Ce^{2t}}.$$

By mimicking the solution of that equation, we can find the solution to this one:

$$\begin{aligned}
\frac{v'}{v-\alpha} - \frac{v'}{v+\alpha} &= -2\alpha k_0 \\
\frac{v-\alpha}{v+\alpha} &= Ce^{-2\alpha k_0 t} \\
v &= \alpha \frac{1 + Ce^{-2\alpha k_0 t}}{1 - Ce^{-2\alpha k_0 t}} \\
v &= \sqrt{\frac{9.8}{k_0}} \left(\frac{1 + Ce^{-2\sqrt{9.8 k_0} t}}{1 - Ce^{-2\sqrt{9.8 k_0} t}} \right)
\end{aligned}$$

Equilibrium solutions at

$$v = \pm \sqrt{\frac{9.8}{k_0}}$$

To solve

$$x' = x^2(x-1),$$

we using the partial fraction decomposition

$$\frac{1}{x^2(x-1)} = \frac{1}{x-1} - \frac{1}{x} - \frac{1}{x^2}$$

which gives

$$\begin{aligned}
x' &= x^2(x-1) \\
\frac{x'}{x^2(x-1)} &= 1 \\
\ln\left|\frac{x-1}{x}\right| + \frac{1}{x} &= t + C \\
\frac{x-1}{x} e^{1/x} &= Ce^t.
\end{aligned}$$

This defines x implicitly as a function of t: there is not much more we can do with it to put it into explicit form.