Review Sheet for Make-up Exam

September 23, 2005

For each of the following situations

(a) Write an initial value problem, giving the meaning of any parameters.

(b) Solve the initial value problem.

1. A population of fish in a pond initially at 1000 grows at a continuous annual rate of 5%, and is harvested at the rate of 50 per year.

   Answer: (a) $x' = 0.05x - 50$, $x(0) = 1000$ (b) $x = 1000$

2. A container of ice cream is removed from a refrigerator at a temperature of 20° and is allowed to come to room temperature.

   Answer: (a) $x' = a(x - b)$, $x(0) = 20$, where b is room temperature and $a < 0$ is the rate of cooling (b) $x = b + (20 - b)e^{at}$

3. A growing population of bacteria, with an initial value of 10^6, is treated with a drug that kills them off at an exponentially decaying rate, initially at 150 per minute.

   Answer: (a) $x' = rx - 150e^{-at}$, $x(0) = 10^6$, where r is the growth rate of the bacteria and a is the decay rate of the drug's effectiveness (b)

$$x = (10^6 - \frac{150}{k+a})e^{rx} + \frac{150}{k+a}e^{-at}$$