Exercises and Problems for Section 0.0

1. You plan to drive 300 miles at 55 miles per hour, stopping for a two-hour rest. You want to know t, the number of hours the journey is going to take. Which of the following equations would you use?

(A) $55t = 190$ **(B)** $55 + 2t = 300$
(C) $55(t + 2) = 300$ **(D)** $55(t - 2) = 300$

In Problems 2–8, both a and x are positive. What is the effect of increasing a on the value of the expression? Does the value increase, decrease, or remain unchanged?

2. $ax + 1$ **3.** $x + a$ **4.** $x - a$

5. $\frac{x}{a} + 1$ **6.** $x + \frac{1}{a}$ **7.** $ax - \frac{1}{a}$

8. $a+x-(2+a)$

In Exercises 9–11,

(a) Write an algebraic expression representing each of the given operations on a number b.
(b) Say whether the expressions are equivalent, and explain what this tells you.

9. "Multiply by one fifth"
"Divide by five"

10. "Multiply by 0.4"
"Divide by five-halves"

11. "Multiply by eighty percent"
"Divide by eight-tenths"

12. To convert from miles to kilometers, Abby takes the number of miles, m, doubles it, then subtracts 20% from the result. Renato first divides the number of miles by 5, and then multiplies the result by 8.

(a) Write an algebraic expression for each method.
(b) Use your answer to part (a) to decide if the two methods give the same answer.

In Problems 13–20, decide for what value(s) of the constant A (if any) the equation has

(a) The solution $x = 0$ **(b)** A positive solution
(c) No solution

13. $3x = A$ **14.** $Ax = 3$

15. $3x + 5 = A$ **16.** $3x + A = 5$

17. $3x + A = 5x + A$ **18.** $Ax + 3 = Ax + 5$

19. $\frac{7}{x} = A$ **20.** $\frac{A}{x} = 5$

Without solving them, say whether the equations in Problems 21–32 have a positive solution, a negative solution, a zero solution, or no solution. Give a reason for your answer.

21. $3x = 5$ **22.** $3a + 7 = 5$

23. $5z + 7 = 3$ **24.** $3u - 7 = 5$

25. $7 - 5w = 3$ **26.** $4y = 9y$

27. $4b = 9b + 6$ **28.** $6p = 9p - 4$

29. $8r + 3 = 2r + 11$ **30.** $8 + 3t = 2 + 11t$

31. $2 - 11c = 8 - 3c$ **32.** $8d + 3 = 11d + 3$

33. A peanut, dropped at time $t = 0$ from an upper floor of the Empire State Building, is at a height in feet above the ground t seconds later given by

$$\text{Height} = -16t^2 + 1024.$$

What does the factored form

$$-16(t - 8)(t + 8)$$

tell us about when the peanut hits the ground?

34. The profit (in thousands of dollars) a company makes from selling a certain item depends on the price of the item. The three different forms for the profit at a price of p dollars are:

$$\begin{aligned} \text{Standard form:} &\quad -2p^2 + 24p - 54 \\ \text{Factored form:} &\quad -2(p - 3)(p - 9) \\ \text{Vertex form:} &\quad -2(p - 6)^2 + 18 \end{aligned}$$

(a) Show that the three forms are equivalent.
(b) Which form is most useful for finding the prices that give a profit of zero dollars? (These are called the break-even prices.) Use it to find these prices.
(c) Which form is most useful for finding the profit when the price is zero? Use it to find that profit.
(d) The company would like to maximize profits. Which form is most useful for finding the price that gives the maximum profit? Use it to find the optimal price and the maximum profit.

Prices are increasing at 5% per year. What is wrong with the statements in Problems 35–43? Correct the formula in the statement.

35. A \$6 item costs $\$(6 \cdot 1.05)^7$ in 7 years time.

36. A \$3 item costs $\$3(0.05)^{10}$ in ten years time.

37. The percent increase in prices over a 25-year period is $(1.05)^{25} \cdot 100$.

38. If time t is measured in months, then the price of a \$100 item at the end of one year is $\$100(1.05)^{12t}$.

39. If the rate at which prices increase is doubled, then the price of a \$20 object in 7 years time is $\$20(2.10)^7$.

40. If time t is measured in decades (10 years), then the price of a \$45 item in t decades is $\$45(1.05)^{0.1t}$.

41. Prices change by $10 \cdot 5\% = 50\%$ over a decade.

42. Prices change by $(5/12)\%$ in one month.

43. A \$250 million town budget is trimmed by 1% but then increases with inflation as prices go up. Ten years later, the budget is $\$250(1.04)^{10}$ million.

44. Match the statements (a)–(c) with one or more of the equations (I)–(VIII). The solution for t in the equation should be the quantity described in the statement. An equation may be used more than once. (Do not solve the equations.)

I. $4(1.1)^t = 2$ II. $(1.1)^{2t} = 4$
III. $2^t = 4$ IV. $2(1.1)^t = 4$
V. $4(0.9)^t = 2$ VI. $(0.9)^{2t} = 4$
VII. $2^{-t} = 4$ VIII. $2(0.9)^t = 4$

(a) The doubling time for a bank balance.
(b) The half-life of a radioactive compound.
(c) The time for a quantity growing exponentially to quadruple if its doubling time is 1.

45. Assume $r > 0$. Without solving equations (I)–(IV) for x, decide which one has

(a) The largest solution
(b) The smallest solution
(c) No solution

I. $3(1+r)^x = 7$ II. $3(1+2r)^x = 7$
III. $3(1+0.01r)^x = 7$ IV. $3(1-r)^x = 7$

46. Assume that a, b, r are positive and that $a < b$. Consider the solution for x to the equation $a(1+r)^x = b$. Without solving the equation, what is the effect of increasing each of a, r, b, while keeping each of the other two fixed? Does the solution increase or decrease?

(a) a **(b)** r **(c)** b

In Problems 47–54, decide for what values of the constant A the equation has

(a) A solution **(b)** The solution $t = 0$
(c) A positive solution

47. $5^t = A$

48. $3^{-t} = A$

49. $(0.2)^t = A$

50. $A - 2^{-t} = 0$

51. $6.3A - 3 \cdot 7^t = 0$

52. $2 \cdot 3^t + A = 0$

53. $A5^{-t} + 1 = 0$

54. $2(0.7)^t + 0.2A = 0$