Math 407
Exam 2
Fall 2004

Name: _____

1	
2	
3	
4	

1. Consider the function $f : A \to B$ given by $f(x) = \cos x$, where A and B are subsets of $\mathbb{R}$.

   (a) Give a choice of A and B that makes f 1-1 and onto.

   (b) Now give a different choice of A from the one you chose in part (a) such that f is still 1-1 and onto.

   (c) Repeat part (a) and (b) with the function $g(x) = x^2 - 2x + 3$.

2. Let $f(x) = 1 - 3x^2$ and $g(x) = 2x^3 + x + 1$.

   (a) Which function has the greater growth rate as x gets large?

   (b) Write the ratio $f(x)/g(x)$ in a form that could be used to explain why your answer to part (a) is true.

   (c) Give the explanation to goes with the expression you wrote in part (b).

3. Consider the following equations for $p, q \in \mathbb{R}$:

$$\begin{aligned} 5p &= 7q \\ 5q &= 7p \\ \frac{p}{q} &= \frac{7}{5} \\ 7q &= 5p \end{aligned}$$

(a) Give all the equations in the list that are equivalent to the first one.

(b) For the other equations, explain why they are not equivalent to the first one.

4. For each of the following equations, (i) find all real solutions to the equation (ii) explain how you could have decided if the equation had any real solutions or not without solving the equation. (You can do (i) and (ii) in any order you like.)

   (a) $2x + 5 = 3 - 6x$

   (b) $\dfrac{3x + 1}{3x + 5} = 1$

(c) $2(x-1)^2+5=4-3(x-2)^2$

(d) $3^{5x-1}=6$