Why are translations isometries? Definition of isometry: the distance between C and C' is the same as the distance between $T_{\vec{AB}}(C)$ and $T_{\vec{AB}}(C')$. Given the geometric definition of $T_{\vec{AB}}$, prove it is an isometry. This can be proved using facts about parallel lines and SAS.

Analytic definition of a translation We represent points as coordinate pairs (x, y), the vector from A to B is (h, k), then

$$T_{\vec{AB}}(x, y) = (x + h, y + k).$$

Definition of isometry: the distance between (x, y) and (x', y') is the same as the distance between $T_{\vec{AB}}(x, y)$ and $T_{\vec{AB}}(x', y')$. The distance bewteen (x, y) and (x', y') is

$$\sqrt{(x' - x)^2 + (y' - y)^2}$$

This is an isometry because the distance $(x + h, y + k)$ and $(x' + h, y' + k)$ is

$$\sqrt{((x' + h) - (x + h))^2 + ((y' + k) - (y + k))^2} = \sqrt{(x' - x)^2 + (y' - y)^2}.$$

Geometric definition of rotation The rotation about the center C through an angle ϕ, denoted $R_{C,\phi}$

Analytic formula

$$\begin{aligned} R_{C,\phi}(1, 0) &= (\cos\phi, \sin\phi) \\ R_{C,\phi}(0, 1) &= (-\sin\phi, \cos\phi) \\ R_{C,\phi}(x, y) &= (x\cos\phi - y\sin\phi, x\sin\phi + y\cos\phi) \end{aligned}$$

$$R_{C,\phi}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$$