Great Issues of Our Time: The Quadratic Formula

William McCallum

Tucson Teachers' Circle, March 26, 2008

A proposal to eliminate quadratic equations

BBC NEWS WORLD EDITION 21 April 2003

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[He advocated] allowing them to drop advanced concepts such as quadratic equations and trigonometry at the age of 14.

The proposal is debated in parliament

The United Kingdom Parliament

26 June 2003

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“Oh dear. I would like to have support from elsewhere as well.”

The minister replies

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- The 16-year-old thought that quadratic equations were logical and fairly straightforward because 'you substitute stuff into a formula'. . . .
- The head of maths said that quadratic equations formed an important step in students' ability to solve equations, . . .

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- The 16-year-old thought that quadratic equations were logical and fairly straightforward because 'you substitute stuff into a formula'. . . .
- The head of maths said that quadratic equations formed an important step in students' ability to solve equations, . . .
- The engineer said that he did not use quadratic equations now, but had in the past . . . "

Al-Khwarizmi, Hisab al-jabr w’al-muqabala, 830

علي تسعة وثلثين ليتم السطح الاعظم الذي هو سطح ره فبلغ
ذلك كله اربعة وستين فاخذنا جذرها وهو ثمانية وهو احد
اضلاع السطح الاعظم فاذا نقصنا منه مثل ما زدنا عليه وهو
خمسة بقي ثلثة وهو ضلع سطح اب الذي هو المال وهو جذره
والمال تسعة وهذه صورته

واما مال واحد وعشرون درهما يعدل عشرة اجذاره فانا
نجعل المال سطحا مربعا مجهول الاضلاع وهو سطح اد ثم نضم
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. . . what is the square which combined with ten of its roots will give a sum total of 39? The manner of solving this type of equation is to take one-half of the roots just mentioned. . . . Therefore take 5, which multiplied by itself gives 25, an amount which you add to 39 giving 64. Having taken then the square root of this which is 8, subtract from it half the roots, 5 leaving 3.

Exercise

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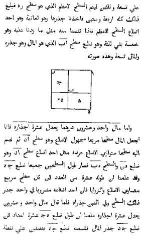
Exercise

What equation is al-Khwarizmi talking about here?

▸ Answer ▸ Skip

Al-Khwarizmi, Hisab al-jabr w'al-muqabala, 830



. . . *what is the square which combined with ten of its roots will give a sum total of 39?* *The manner of solving this type of equation is to take one-half of the roots just mentioned. . . . Therefore take 5, which multiplied by itself gives 25, an amount which you add to 39 giving 64. Having taken then the square root of this which is 8, subtract from it half the roots, 5 leaving 3.*

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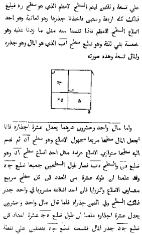

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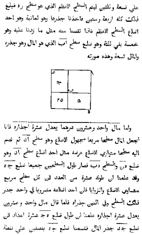

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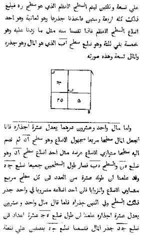

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$$x^2 + 10x = 39$$

$$x^2 + 10x + 25 = 39 + 25 = 64$$

$$x + 5 = 8$$

$$x = 3$$

Al-Khwarizmi's geometric proof of his method

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\begin{aligned}
x^2 + 10x &= 39 \\
x^2 + 10x + 25 &= 39 + 25 = 64 \\
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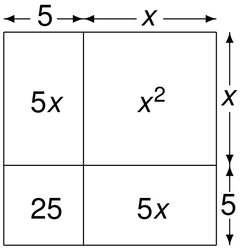

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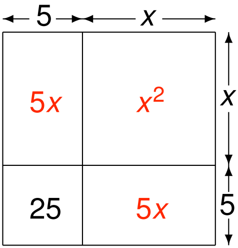

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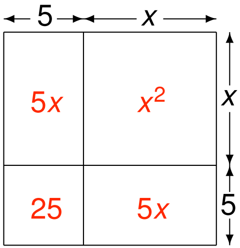

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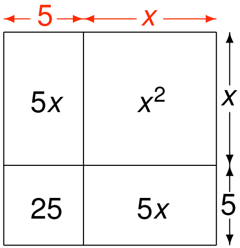

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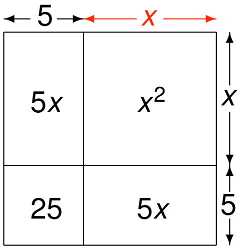

Exercise

Al-Khwarizmi's geometric proof of his method

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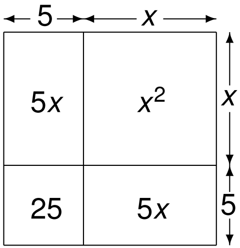

Exercise

Draw a diagram that illustrates the solution of the equation

$$x^2 = 39 + 10x.$$

Answer Skip

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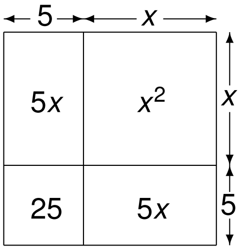

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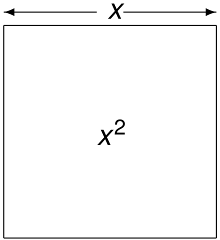

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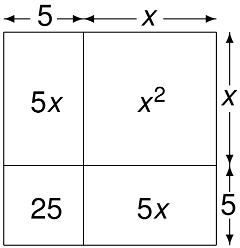

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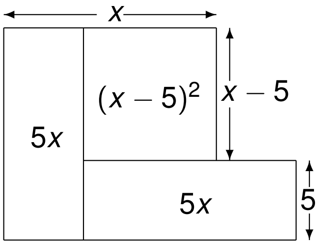

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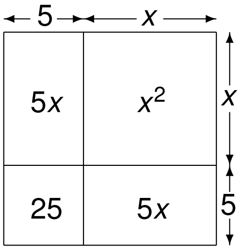

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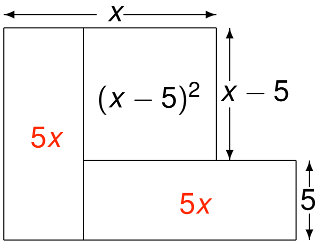

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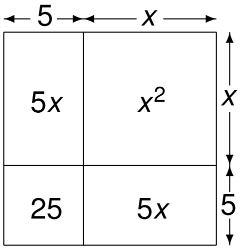

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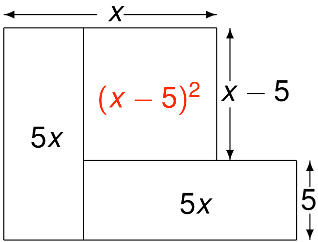

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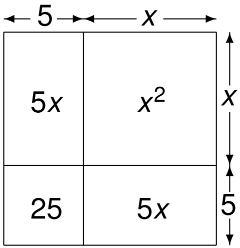
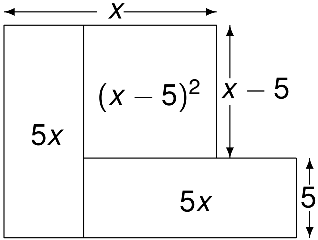


$$\begin{aligned} x^2 &= 39 + 10x \\ x^2 + 25 &= 64 + 10x \\ x - 5 &= 8 \\ x &= 13 \end{aligned}$$



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- So each equation must have a negative solution as well.
- Imaginary solutions were far in the future.

Completing the square the modern way

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x^2 + bx + c &= 0 \\
x^2 + bx &= -c \\
x^2 + bx + \left(\frac{b}{2}\right)^2 &= -c + \left(\frac{b}{2}\right)^2
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x &= \frac{-b \pm \sqrt{b^2 - 4c}}{2}
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\begin{aligned}
ax^2 + bx + c &= 0 \\
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\left(x + \frac{b}{2a}\right)^2 &= \frac{b^2 - 4ac}{4a^2} \\
x + \frac{b}{2a} &= \pm\frac{\sqrt{b^2 - 4ac}}{2a} \\
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\end{aligned}
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The quadratic formula

A number x satisfies

$$ax^2 + bx + c = 0, \quad a \neq 0,$$

if, and only if,

$$x = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad \text{or} \quad x = \frac{-b - \sqrt{b^2 - 4ac}}{2a}.$$

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Exercise

Give some simple conditions on the coefficients for a quadratic equation to have

(a) two real roots ▸ Answer

(b) two positive roots. ▸ Answer

▸ Examples ▸ Skip

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if, and only if,

$$x = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad \text{or} \quad x = \frac{-b - \sqrt{b^2 - 4ac}}{2a}.$$

Answer

(a) Make a and c have opposite signs.

Answer to (b)

Examples

The quadratic formula

A number x satisfies

$$ax^2 + bx + c = 0, \quad a \neq 0,$$

if, and only if,

$$x = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad \text{or} \quad x = \frac{-b - \sqrt{b^2 - 4ac}}{2a}.$$

Answer

(a) Make a and c have opposite signs.

(b) Make a and c have the same sign, and make b negative but large in magnitude.

Examples

Another way of solving quadratic equations

For all numbers x,

$$x^2 + 10x - 39 = (x - 3)(x + 13) = 0.$$

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The product of two numbers is zero if, and only if, one of them is zero, so either

$$x - 3 = 0 \quad \text{or} \quad x + 13 = 0.$$

That is, $x = 3$ or $x = -13$.

Viete's formulae

Theorem (Viete's formulae)

The numbers r and s are the solutions to

$$x^2 + bx + c = 0$$

if, and only if,

$$r + s = -b \quad \text{and} \quad rs = c.$$

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Proof of the "if" part.

If $r + s = -b$ and $rs = c$, then

$$x^2 + bx + c = (x - r)(x - s).$$

Then the argument goes as in the previous slide. Exercise □

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Exercise

How do you prove the "only if" part? That is, if r and s are the solutions to $x^2 + bx + c$, then $r + s = -b$ and $rs = c$.

Answer Skip

Viete's formulae

Theorem (Viete's formulae)

The numbers r and s are the solutions to

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Proof of "only if" part

Divide $x - r$ into $x^2 + bx + c$, so

$$x^2 + bx + c = (x - r)q(x) + R.$$

Putting $x = r$ we get $R = 0$. Then $q(x) = x - t$ for some t, and the only possibility is $t = s$.

Viete's formulae and the quadratic formula

If

$$x^2 + bx + c = 0$$

then let

$$r = \frac{-b + \sqrt{b^2 - 4c}}{2} \quad \text{and} \quad s = \frac{-b - \sqrt{b^2 - 4c}}{2}$$

Exercise

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Exercise

Give an explanation, purely in terms of the structure of the expressions, of why these two numbers satisfy

$$r + s = -b \quad \text{and} \quad rs = c.$$

▸ Answer ▸ Skip

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Answer

When you add r and s, the plus and minus signs cancel.

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Viete's formulae and the quadratic formula

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$$x^2 + bx + c = 0$$

then let

$$r = \frac{\square + \boxed{}}{2} \quad \text{and} \quad s = \frac{\square - \boxed{}}{2}$$

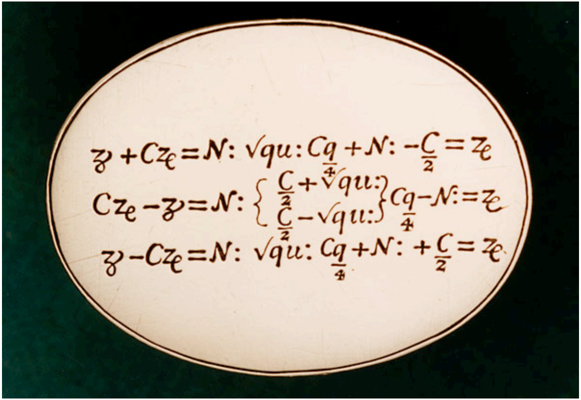
Answer

When you add r and s, the plus and minus signs cancel.

When you multiply r and s, you get the difference of two squares in the numerator,

$$(-b)^2 - (\sqrt{b^2 - ac})^2 = b^2 - (b^2 - 4c) = 4c.$$

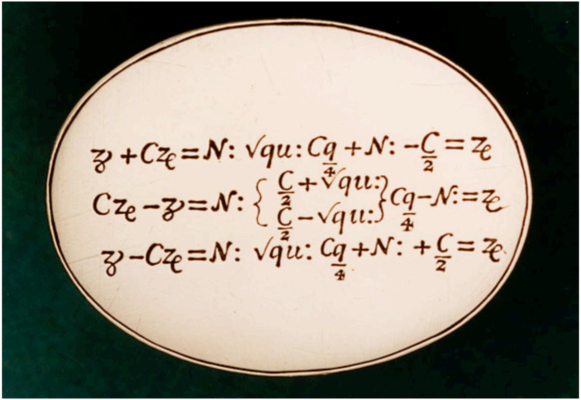
The quadratic formula in the 17th century



From the Oxford Museum of History of Science (Stephen Johnston, photo Bluebridge Farm Studio)

Exercise

The quadratic formula in the 17th century



Exercise

What is going on here?

Answer

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$$z + Cr = N : \sqrt{qu} : \frac{Cq}{4} + N : \; - \frac{C}{2} = r$$

$$Cr - z = N : \left\{ \begin{array}{l} \frac{C}{2} + \sqrt{qu} : \\ \frac{C}{2} - \sqrt{qu} : \end{array} \right\} \frac{Cq}{4} - N : \; = r$$

$$z - Cr = N : \sqrt{qu} : \frac{Cq}{4} + N : \; + \frac{C}{2} = r$$

What is going on here?

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What is going on here?

$$x^2 + Cx = N, \qquad \sqrt{}qu : \frac{C^2}{4} + N : -\frac{C}{2} = x$$

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What is going on here?

$$x^2 + Cx = N, \qquad \sqrt{\frac{C^2}{4} + N} - \frac{C}{2} = x$$

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$$x^2 - Cx = N, \qquad \sqrt{\frac{C^2}{4} + N} + \frac{C}{2} = x$$

Example

If

$$x^2 + 10x - 39 = 0,$$

then

$$x = \frac{-10 \pm \sqrt{256}}{2} = -5 \pm \sqrt{64} = 3, -13.$$

Example

If

$$x^2 - 10x + 9 = 0$$

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$$x = \frac{10 \pm \sqrt{10^2 - 4 \times 9}}{2} = \frac{10 \pm \sqrt{64}}{2} = 5 \pm 4 = 1, 9.$$

◂ Back to exercise

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